Fractal Dimension in Architecture: An Exploration of Spatial Dimension

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To the named clouds

(for always being the source of inspiration)

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DECLARATION

Herewith I declare that I have prepared this Master thesis independently, that it has not been submitted in the same or similar wording as an examination paper in another course of study, and that I have not used any other aids and sources than the ones indicated.

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ABSTRACT

Mandelbrot (1975) coined the term, *Fractal* to define natural forms and the hidden but simple code behind their seemingly complex appearance. Fractal dimension, the rudiment of fractals, usually described as space filling property, provides a foundation for analyzing roughness of surfaces.However, roughness exists in all dimensions. Architecture, innately related to geometry/forms, shows roughness at many scales from urban fabric to a building's facades and plans. Several types of research have concluded that the fractal dimension of facades and plans shows levels of formal information spread in several scales. Moreover, it is argued that fractal analysis of a plan elucidates the experience of space. In the context of architecture, this experience is highly important to design livable space. However, existing measurement system i.e box counting method, widely used in the architectural fractal analysis, provides a fractional dimensional value between 1 & 2 providing information on the planar form. On the contrary, the architecture is about space i.e. 3-Dimensions. It is likely that *space* is better understood with higher fractal dimensional analysis. But, it is important to correlate 2D and 3D fractal analysis from their basic definition i.e. the theory of fractal. For instance, a smooth surface (2D-dimensional value) becomes rough (<2D-fractal dimensional value) by puncturing holes in it (eg. Sierpinski Triangle). This is a reverse thinking of making fractals. Space is formed by putting many rough or smooth surfaces (like walls, floor, roof and so on) together in a variety of ways. Again, in reverse thinking, in the 3-dimensional world, an empty space we live in is the hole in the 3D and understanding this hole with 2-dimensional fractal analysis seems deficient. Therefore, this observation led to the research question: How fractal dimension on 2D planes limits our understanding of the spatial/3D world? How can we measure 3D fractal dimension? Based on the observational analysis of past research in the field of fractal analysis in architecture (Michael Batty and Longley, 1994, Carl Bovill, 1996, Nikos A. Salingaros, Michael J. Ostwald and so on) and exploration of existing 3D fractal dimension measurement systems in fields like cardiology, neuroscience and their relevance in architecture is the main research for this dissertation. In addition, a new way based on the theory of fractals to understand and possibly to measure 3D fractal dimension i.e. Un-Folding space method, a rather analytical and observational approach to understand higher fractal dimension, will be devised and explained as the next step towards achieving higher fractal dimension analysis in architecture.

Keywords: Fractal geometry, 2D and 3D Fractal dimension, levels of scale, Unfolding space method, space

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1 INTRODUCTION

'Nothing exists except atoms and empty spaces; everything else is opinion'[1]

[Democritus, 400B.C]

Humans have been amazed by the nature's way of creating forms which seems incomprehensible because of its chaotic and complex appearance. However, Mandelbrot (1982) showed how it is important to look beyond what is seen at first glance , to search for what created these complex forms, the code behind the complex appearance and not to be overwhelmed by the result or final geometry. In the era when every form is considered to be smooth and of Euclidean geometry which dictated more than 2400 years in every kind of field that incorporates geometry or forms, it was a revolutionary idea to introduce geometry of roughness. The classical Euclidean geometry is limited in defining many of the natural forms and phenomena. Mandelbrot's constant query regarding the natural forms not satisfied by these smooth forms and consistent effort with effective solutions made us see the true code hidden behind nature's complexity. Though several mathematicians and geographers had understood the limitations of Euclid's geometry, none of them actually could give the solutions to the limited classical geometry. It was the time of modern computers in 1980s which made it possible for Mandelbrot to visualize the mathematical solutions of these problems in the pictures or graphics what previous generation of mathematicians could not do. The story began in 1958 when the giant corporation of modern technology, IBM was seeking for creative thinkers and mathematicians (fractal, 2011/youtube). Mandelbrot soon quit the job in France and joined the research lab in IBM. The first problem reported to the group of mathematicians in the research lab was regarding the transfer of computer data through cables over phone lines. The process of transferring, sometimes, could not succeed i.e the data does not go through (profile 2017). This created a lot of noise while transferring data. To resolve the issue the noise data was graphed, Mandelbrot found out the

1 https://www.brainyquote.com/quotes/quotes/d/democritus384195.html

similarity feature of graph irrespective of time. The data graphed for one day, for one hour and even for one second looked exactly the same (fractal, 2011/youtube).

Fig. 1 Noise plot in different time scale

This reminds him of the mystery of the monsters which he confronted as a young mathematician in France. These monsters were an unsolved puzzle for all the mathematicians. These were such mathematical formula or a method of execution which exhibited weird shapes and paradoxical in nature which can not be graphed and understood fully (profile 2017). Kantor set, Peano curve, Koch curve are few examples of such monsters which are explained in Chapter 2. All of these monsters shows the self similarity irrespective of how much one zooms in or out. Mandelbrot was confronted with the same phenomena in the noise graph. The typical example of Koch curve, known as pathological curve, where one thinks the length to be finite in the given shape finds the length to be infinitely long is exactly related to the problem of length of coastline measurement. The length of coastline varies according the stick used, shorter the stick longer will be length measured. This particular phenomena is described in Chapter 2. This led Mandelbrot to suspect that something else can be measured rather than length in these cases. He named it roughness. This is the point of departure for the geometry of roughness. The measurement of this roughness is the fractal dimension. Higher the value of fractal dimension more rougher the surface is. In nature everything is rough from clouds, plants, mountains, borderlines, trees, weather patterns, growth of cities, growth of population, economic system, formation of stars and galaxies, surfaces of moon, mars, earth like bodies in the space to the nerves in human body, bronchi of the lungs, neurons in the brain up to our heartbeat rate. Suddenly all these roughness came into view which was hidden or not understood even if seen. Since this new geometry, fractal geometry, which Mandelbrot considers an extension to extant Euclidean geometry, encompasses such a wide range of fields from arts, biology, human anatomy, economics, ecology, demographics to astronomy, almost every field has experienced new comprehension and applications in their respective fields. Architecture, on the other hand, is another field which is deeply rooted in its history, culture, society, philosophy, economics and stylistic movements. Several movements like classical, modernism, post-modernism, deconstructivism in the architecture fraternity have risen and fallen. However, in connection with new geometry of fractals, I would like to take a different point of seeing the issue. Generally, the usual notion of comprehending relies on seeing and understanding the meaning of it. In architecture, physical structures and spaces are understood mostly in semiotics (the study of symbols, signs and their interpretation). The semiological Triangle (a triangular relationship among perception, conception, and representation) proposed by Charles Jencks (1969) clarifies, „...[in the context of] architecture, one sees the building, has an interpretation of it and usually puts that into words ... In most cases, there is no direct relation between a word and a thing, except in the highly rare case of onomatopoeia" (the formation of a word from a sound associated with what is named (e.g. *cuckoo*, *sizzle*)[2]. Jencks points out an important point of existence of relations or correlations between language, thought, reality . It is not necessary for one area to determine the other. In architecture alone, metaphors have been used to correlate contextual reading of the forms given to a building such as Circular dome signifying infinity and timelessness, Triangle, in Christianity representing, God as Trinity of Father, son and the Holy Ghost and so on. However, if one accepts the above quote, it is essential to strip-down all the opinions and metaphors from

2 http://jomichaels.blogspot.de/2013/02/on-onomatopoeias.html?showComment=1361200102769

the body of architecture and make the invisible visible where the actuality of architecture lies. The hidden complexity and chaos, order in randomness, nonlinearity and dynamic nature of architecture, rarely addressed so far, is the actuality of architecture. In the context, as cited by Batty & Longley 1994, relativity, quantum mechanics and chaos being the most memorable inventions of last century (James Gleick, *Chaos 1987),* it is essential to open up the uncertainty and complexities involved in architecture whereas conventional architectural practices advocate for the certainty and rationality. Jencks (1997) argues the ordered complexities[and chaos], processes of the universe, its growth, sudden leaps, and twists can be reflected in Architecture. This dissertation is simply trying to address the basic notion of fractal geometry i.e. fractal dimension and its limitations in the existing systems.

1.1 Context

The form language expressed in geometry is an essential part of architecture along with many other theories on which architecture touches upon. Though the concept of fractals is around for more than four decades, the power of its applications in the field of architecture is explored at its least. The actual concept of fractals in relation with complexity theory is just realized in symbolic and metaphorical level in architecture. Therefore It is essential to decipher the terms like Generative designs, the scale of hierarchy, form language, pattern language and fractals to its basic notion. For example, a cell as the basic unit of life reveals secrets of a living body. In the context of application of fractals, understanding fractal dimension, its limitations and further research are necessary.

1.2. Research Aim and Objectives

The research aim is to open up the contemplative descriptions of spatial fractal dimension, its importance in understanding space. A theoretical approach for improving the existing method is the major objective. Few other objectives of this research are:

- To analyze the connection between fractals and architecture in symbolic or metaphoric level.

- To comprehend the existing systems of fractal dimension measurement from urban analysis up to a house
- To contemplate and discuss the spatial feature in terms of fractals
- To remedy the loopholes in existing systems to better understand fractal dimension through an exploration of spatial dimension

1.3. Scope of Research

The departure of study is theoretical observations of existing fractal dimension measurement systems and its limitations whereas the theoretical underpinning/establishment of new approach or improvements in the existing systems will be the scope. The research is limited in providing actual applications or software in improved measurement system since it is just a theoretical exploration.

1.4. Method and structure

The methodology followed for this research is based on an analytical approach to existing research papers and comparative studies along with personal communication and interviews with the professors and researchers in the field of Fractal dimension measurement. Hence, the outcome is an observational finding.The structure of the dissertation is as follows;

Chapter 2 A theory of Fractal sheds light on the evolution of fractal as a new geometry of roughness. It points out the major mathematicians and their successive contributions to the foundation of Fractals. The chapter also includes sub-chapters explaining the relationship between the science of complexity and fractals as well as a comparative overview on ideal/mathematical fractals and natural fractals.

Chapter 3 Fractals and Architecture mainly outlines the idea of fractals in architecture. The first sub-chapter is an overview of how fractals were then and are now in architecture. It provides the overall scenario of present-day research status on the descriptive level. How is fractal introduced by architects like Peter Eisenman in architecture? How was it received by architecture community? Why it declined as the time passed by? These points are summarized. The second sub-chapter commences with the idea of Christopher Alexander's Pattern language and its relation with fractals from Nikos A. Salingaros's perspective. It also clarifies the need of *Form language* in addition to *pattern language* to make the dead urban design alive again. The third sub-chapter illustrates the relationship between topology and architecture.

Chapter 4 Measuring Architecture begins with the basic notion of dimension that defines architecture, why is dimension important? What can dimensional analysis provide? Then the second sub-chapter opens up a basic definition of dimension and how it changes in case of non-smooth shapes like coastlines, mountains, clouds and so on. This new path leads to the Box-counting method, which is our major concern for the research. The subsequent subchapter illustrates the details needed for fractal analysis. Why framework and refinement of the images before analysis are described briefly based on Ostwald and Vaughan's book. The sub-chapter, use of box counting dimension in architecture' explores several analysis done by several architects and researchers starting from Michael Batty and Paul Longley (1994), Carl Bovill(1996), Michael J. Ostwald, Wolfgang Lorenz, Jon Cooper, Ron Eglash and so on specifically based on topics of accessing urban character, Urban growth analysis, study of visual complexity of facades and plans and so on.

Chapter 5 Spatial Fractal Dimension commences with an evaluation of successive improvements in FD measurement systems and their present state. A perspective fractal analysis, taking the analysis of Robbie house conducted by Ostwald & Vaughan (2016) is described and evaluated. The existing system of cube counting or voxelization in case of trabecular bone imaging, tree models, and brain MRI is explained briefly. Then the subsequent chapter reflects the contemplative thought on space and how space is manipulated or understood along with describing how one can measure 3DFD. The ‚Un-Folding Space Method' is put forward as a thought rather than a specific method along with other few methods.

Chapter 6 Conclusion and Discussion is a contemplative summary of dynamic nature of spatial fractal dimension. Beginning from perception, the discussion goes towards the experience of space with fractality. It also sheds light on the probable future research and methods needed for better comprehension.

2 THEORY OF FRACTALS

In mathematics, a function is a relation between a set of inputs and based on which a set of output is obtained. Sometimes function is considered as a machine (Fig. 2), say f, which takes input as, x, and results in the output as f(x).

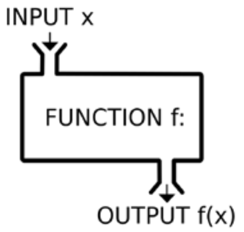

Fig. 2 working function diagram

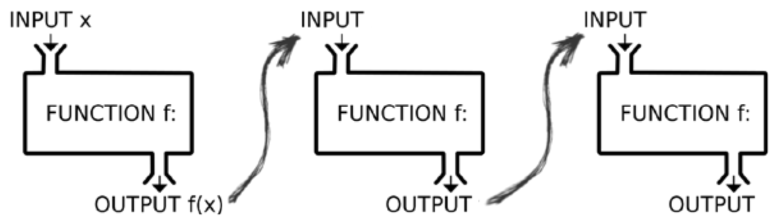

Fig. 3 Iterative function diagrammatic expression

A function can be represented in many forms such as an equation, algorithm or formula, a graph of a function, differential equation or inverse to another function. One interesting thought on function is, what happens if the first output is used as input for the second time in the same function and repeat the process with each output infinitely. This iterative process is given the name as the iterative function. It is the starting point of new mathematics; Fractals.

2.1 History of Evolution of Fractals

Theory of fractal in its present form is not the result of only one mathematician's thinking. That is why it is imperative to reverse ourselves in the timeline to understand the evolution of fractals.

2.1.1 Pierre Fatou (1878-1929), Lewis Fry Richardson (1881-1953), Gaston Julia (1893-1978) and Benoit Mandelbrot (1924-2010)

The structure of the universe is complex...science and art both attempt to explore this in order to understand and then make use of it... maths and science seek to analyze and experience while art attempts to synthesize experience.

M. Fowler, 1996

Initial concepts was developed in the 17th century when the mathematician and philosopher Gottfried Leibniz was pondering about recursive self-similarity. Such recursivity was considered as mathematical monsters. In 1917, a French mathematician Pierre Fatou started working on iterative function which give rise to complex outputs in graphical representations. During the same time, Gaston Julia wrote a paper on rational function "*Mémoire sur l'itération des functions rationnelles*" which gained immense popularity and Julia received the Grand Prix award (*Memoir* translated by Rosa, 2001). However graphical representation of the recursive function for infinite number of times is impossible until the advent of modern-day computers in the 1980s. Another important study in the case of nonlinearity of natural shapes like coastlines by Richardson is important to discuss. Gonze (*Fractals: theory and applications)* notes,

> While studying the causes of war between two countries, Richardson (1961) decided to search for a relation between the probability of two countries going to war and the length of their common border. While collecting data, he realized that there was considerable variation in the various gazetted lengths of international borders. For example, boarder between Spain and Portugal was variously quoted as 987 or 1214 km while that between the Netherlands and Belgium as 380 or 449 km.

This relationship between scale and length of coastline is an essential concept for Mandelbrot to understand scale-invariance and later on developing fractals as new mathematics.

On the extension of Richardson's work Mandelbrot published a paper *How long is the coast of Britain? Statistical self-similarity and fractional dimension* (*Science*, 1967) where the concept of fractional dimension is described explaining the limitation of linear length measurement system being insufficient to describe geographical curves. The quantity other than length is the fractional dimension which provides roughness of the boundary line (in case of coastlines).

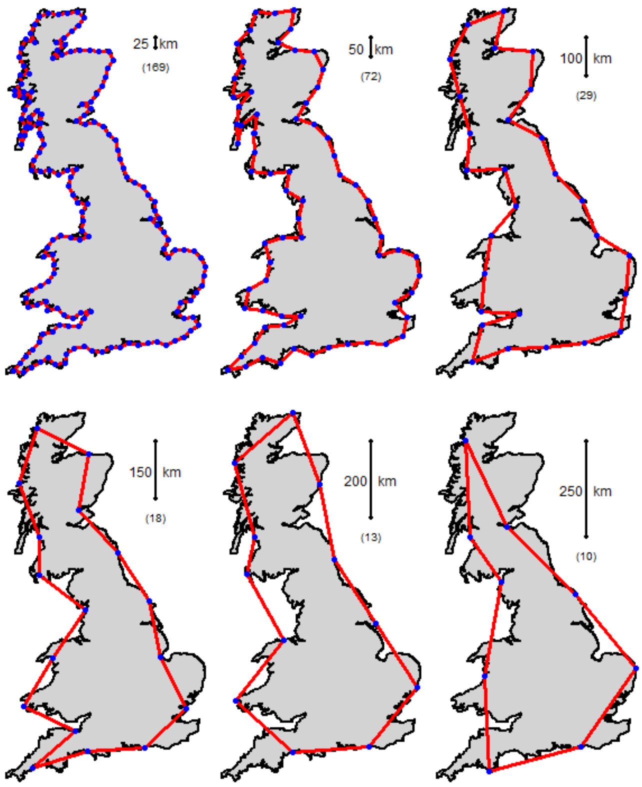

Fig. 4 Coastline of Britain measurement with different length sticks

The paper represents the concept of self-similarity which is defined as "... each portion can be considered a reduced-scale image of the whole" (Mandelbrot, 1967; 636-38). The process of measuring Britain's coastline with a 200km unit (stick) gives rise to approximate 2400km of coastline length, with a 100km Unit gives rise to 2800km and with a 50km stick gives rise to approximate 3400km of coastline (Avsa, 2006). The notion of length in jagged curvilinear shapes like a coastline is dependent on the scale of the map used. This variation in length handicapped geographers to understand the coastlines. The inquiry of what is it that does not change in the case of jagged or rough surfaces like coastlines. It led Mandelbrot to dig deep in the classical geometry and existing dimensioning system. Some other forms of a dimensioning system are needed (Mandelbrot, 1967) to measure this roughness which is not addressed so far. Mandelbrot concretized "the so far esoteric concept of a 'random figure of fractional Dimension'" (Mandelbrot, 1967). The fractal dimension is defined as the ratio of the logarithm of changes in details (in case of coastline, its length) with regards to the changes in scales (different units of measurement). This ratio gives the roughness of the map or object being observed. For instance, Britain's coastline has Fractal dimension, D of 1.25. (ibid). The detailed overview of the origin of fractal dimension is described in Sub-Chapter 4.2 Dimension: A notion.

Benoit Mandelbrot published his first book on fractals in1975 *Les Objects Fractals: Form Hasar et. Dimension* (Ostwald & Tucker, 2007) whose English version was published in 1977, *Fractals: Form, Chance, and Dimension*. The word fractals coined by Mandelbrot literally means fractured, fragmented. Though *The Fractal Geometry of Nature*(1982) can be considered as the pioneer assemblage of concepts, mathematics, and graphics showing how fractal geometry exist in nature from time immemorial. For the first time after the invention of silicon chip in the 1970s, in 1980s Mandelbrot program his simple equation $Z=z^2+c$ which covered the all of Julia set and a graphical representation of Mandelbrot set was published (AllahUniversal79, 2011). Since then the Mandelbrot set became the emblem of fractal geometry. In the book, Mandelbrot elaborated the concept of self-similarity and fractal dimensions found in several of nature's creations, from Snowflakes to Koch curves, from trees structures to constellations of stars and so on. To devise the use of dimensions in fields other than pure mathematics, Mandelbrot applied the fractals in defining nature, "establishing D in a central position in empirical science, thereby showing [use of fractals] to be of far broader import than anyone imagined" (Mandelbrot 1982; 16).

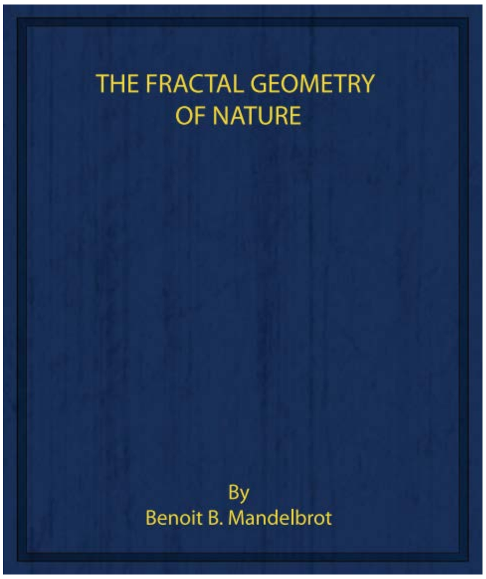

Fig. 5 Book cover, *The Fractal Geometry of Nature,* 1982

2.1.2 Mathematical Fractals and Natural Fractals

Mathematical Fractals

The geometric shapes that show exact self-similarity on infinite scales are considered ideal fractals. These exist only in mathematical theories and computer generated graphics. With the invention of computers, one can generate exactly self-similar fractals. Some of the initial ideal fractals are Van Koch curve, Sierpinski triangle. These fractals were developed in theory long before anybody can see it in actual computer simulated graphics in the 1980s. However, these shapes, even though only in theory, intrigued mathematicians for long. That's why these forms or shapes or say theories were called ‚Monsters' or ‚pathological curves'.

Before taking some examples of these mathematical monsters, it is essential to define few of the terms and how they are integrated into the process of developing the shapes. The starting image is called ‚initiator'. It means any shape from classical geometry can be taken to initiate the process. The second image generated using one simple rule is called ‚a generator'[3]. This rule can be anything like dividing a line into 4 pieces and deleting 2 of them or adding one line to the existing 4 pieces and so on. This rule is the important aspect which governs rest of the process as the same rule or algorithm is repeated again and again up to infinity to achieve the complex ideal fractal images and surfaces.

1. Cantor set:

Since it is a disconnected set, its topological dimension is 0. It has a self-similar property with a fractal dimension of 0.631. Each smaller version, when magnified by three, will give the previous scale original shape (Mandelbrot 1982; p. 80).

3 For a detail overview: Lorenz 2002, Fractal and Fractal architecture, Diploma thesis, p. 16

Fig. 6 Cantor set

In case of Cantor set, the initiator is a straight line. It is divided into three equal segments and the middle one is removed. This first iteration is known as the generator. A similar procedure is applied to the two lines again and again. In general, Cantor set consists of 2^n subsets each with a magnification factor of 3^n. So the fractal dimension of this set is calculated as:

$D_f = \log(2^n)/\log(3^n)$

Or, $D_f = n\log(2)/n\log(3)$

≈ 0.6309

2. Van Koch Curve:

In the Koch curve, the initiator is a line. The straight line is divided into 3 segments and central line is taken away by placing two lines with equal lengths making a supposed equilateral triangle (but without the base line in the middle portion). This shape is the generator. Now the similar procedure is repeated for the four lines. On several iterations, the actual Koch curve is achieved (Mandelbrot 1982; p.42-44). Koch curve is a paradox, to the eyes it seems perfectly finite yet the length of the curve is infinitely long which can not be measured. This so called pathological curve is exactly similar to the coastline paradox.

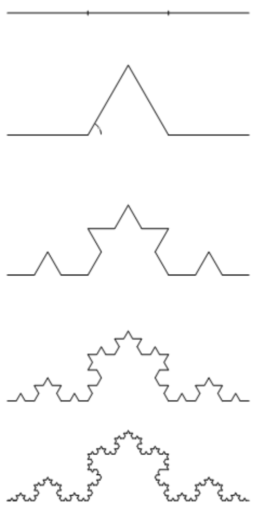

Fig. 7 Koch Curve

With 4^n subsets and a magnification factor of 3^n for n iterations, the fractal dimension of Koch curve is calculated as follows:

$D_f=\log(4^n)/\log(3^n)$

Or, $D_f=n\log(4)/n\log(3)$

≈ 1.261

3. Sierpinski triangle

It is created using triangles. An equilateral triangle is an initiator. Then the simple rule to follow is to remove the central triangle made out of joining middle points of the three sides. The shape so produced is our generator. A similar procedure is followed for every equilateral triangle thus formed for an infinite number of times (Mandelbrot 1982; p.142). On each iteration, 3 subsets are created with a magnification factor of 2. So for n iterations, it will be 3^n and 2^n respectively. The fractal dimension is

$D_f=\log(3^n)/\log(2^n)$

Or, $D_f=n\log(3)/n\log(2)$

≈ 1.585

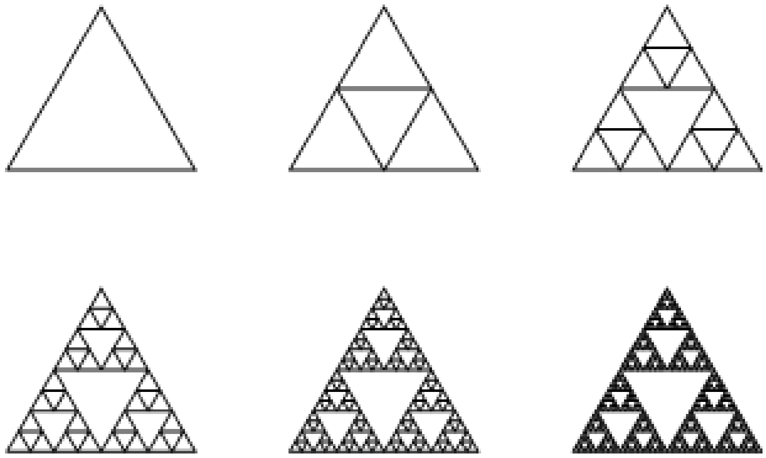

Fig. 8 Sierpinski Triangle successive iterations

4. Mandelbrot set:

Mandelbrot set is an iterative function where the output is put into the formula again and again. Mandelbrot's $Z=z^2+c$ is equally as simple as Einstein's $E=mc^2$ where z represents numbers and coordinates whereas E,m & c represents physical quantities.

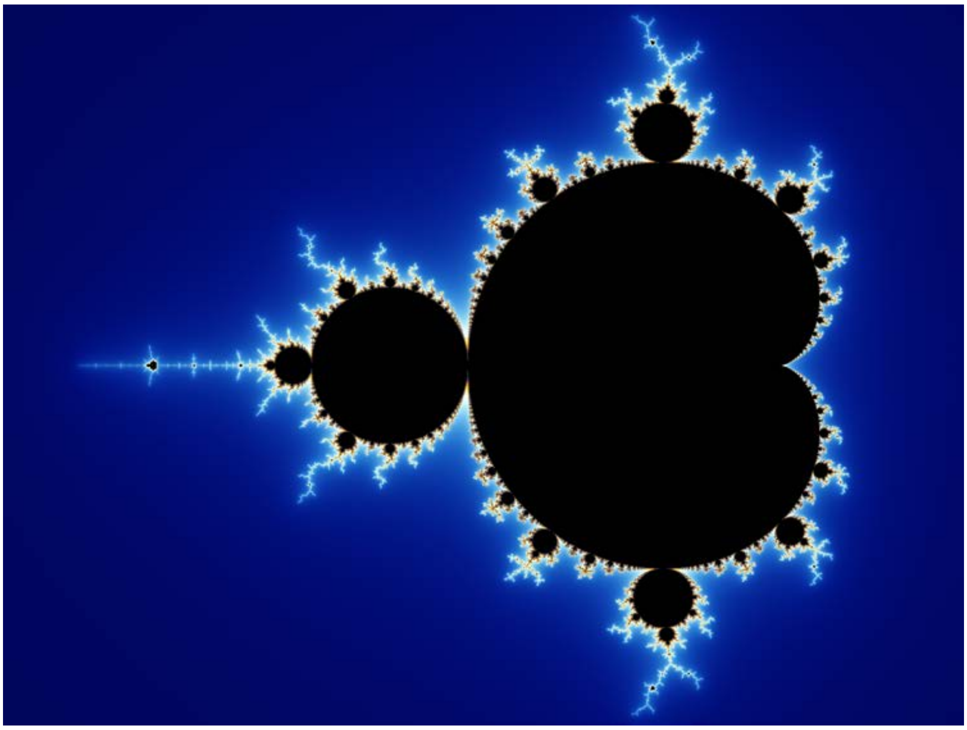

Fig. 9 Mandelbrot set; rendered image

Natural Fractals

Natural lines, surfaces, and objects are not shapes as defined by Euclidean geometry. Euclidean geometry describes all shapes with lines, smooth planes, cones, spheres and cylinders, anything that does not fulfill the definition is considered as ‚noisy Euclidean geometry'. The central role played to resolve this problem is by fractal geometry.

The notion of self-similarity in natural shapes and objects sheds light in understanding natural geometry. The more one zooms in more similar features are observed. However natural fractals are never exactly self-similar as explained in mathematical fractals. These natural fractals show statistical self-similarity which means as one zooms in, statistically/approximately self-similar shapes emerge. Trees, clouds, mountain, coastlines and almost every natural forms show this characteristic.

Fig. 10 Fractals in a Tree

2.2 Science of Complexity and Fractals

The complexity science is a paradigm shift in linear Newtonian cause and effect world. It deals with real life dynamic situations that deal with feedback loops, wicked problems and nonlinearity in operation. The real world runs on several connected components which interact with each other in a system to generate nonlinear dynamics as the outcome. According to Boeing (2017), this rich behavior, the complexity, applies to both dynamics(i.e. processes) and structure (i.e. patterns and configurations) which confirms incrementalism and wicked problems whereas problematizes certainty and rationality. Mandelbrot describes the science of complexity and Fractal are overlapping sets which mean some of their characteristics are similar to each other yet they have many different characteristics which are deviating from each other.

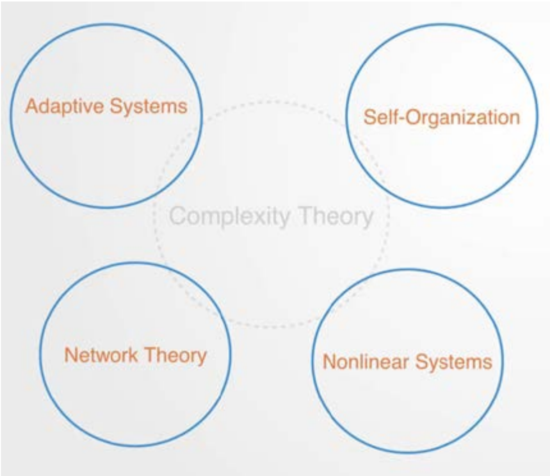

Fig. 11 Overlapping sets in Complexity theory

Jiang (2016) argues that theories like central place theory, Zipf's law, the theory of centers are considered as points of view of complexity theory in Urban design, in sharp contrast to Euclidean geometry and Gaussian statistics which essentially deals with regular shapes and functions. Several architectural theorists from Jencks, Lynn to Eisenmann attempted to apply complexity theory in architecture but their endeavor penetrated only up to diagrammatic and iconic sketchy descriptions. It shows the deficiency in understanding in reality what complexity refers and how it can be dealt with. The post-Jane Jacobs (1961) notion of ‚organized complexity' is accepted by many scholars to define urban morphology and city forms. Based on the premise, a whole is greater than the sum of its parts, several tools have been developed such as complex networks (Newman et al. 2006) and fractal geometry (Mandelbrot 1982) which enable our understanding of complexity in a city as a complex system (Jiang 2016). On the other hand, the living geometry proposed by Alexander (2002-2005) aims for creative process whereas Fractal geometry is the means to attain it. This shows the interdependency of two sciences to reveal the actual nature of systems in our everyday world from finance to eco-system and so on.

3 FRACTALS AND ARCHITECTURE

This chapter describes the details of fractals and its relation to architecture with different hypotheses and their applications. The first sub-chapter is an expedition into the philosophical foundations of complexity science along with fractals in architecture initiated by few architects and the rise and the fall in subsequent decades whereas second and third subchapters will delve into the interdependency of pattern language and fractals to create sustainable cities from Salingaros's perspective and relation between topology and architecture respectively.

After the word Fractals and self-similarity came into existence; its widespread use in every field shadowed the need to understand one major difference between two seemingly same words i.e. Fractal Geometry and Fractal dimension. However, it is essential to distinguish them. Fractal geometry defines a particular set of objects exhibiting a high level of self-similarity. Peitgen & Richter (1986) explains, as cited by Ostwald (2013, p.648), "… fractal geometry is defined by a repetitive or iterative feedback structure that produces a type of deep geometric phenomena known as scaling or characteristic irregularity." Moreover the concept of self-similarity lies in the concept of scale as Kayle (1989) posits, "Scaling is the property by which a figure, when examined at increasingly fine scales, is seen to be self-similar; or that, at a variety of ranges, the object in question tends to resemble itself" (ibid, p. 648)). Both of the definitions emphasize on the fractal geometry being geometric phenomena i.e. iterative feedback structures with details in every scale for example clouds, trees, cauli-flower etc. So it is mainly concerned with a geometric shape. On the flip side of the coin, Manning (1956) describes dimension, as cited by Ostwald (2013, p.647; Ostwald & Vaughan 2016; p. 8), as "a topological measure of the space-filling properties of an object." In the same line, fractal dimension can be understood as the property of the fractal geometry (irregular objects/shape) that fills up space. Faegre (2004) describes a simple relation between lengths measured to the measuring stick used (to measure the total length of the perimeter of the building) identifies a characteristic dimension called fractal dimension. This is in exact correlation with how Mandelbrot measured the coastline of Britain. The comprehension of difference is important for built environment studies as Jencks (1995) situates,

> For Mandelbrot, any set may have a fractal dimension, but only sets with a defined scaling pattern can be described as instances of fractal geometry. This distinction is a critical one in architectural analysis where the two are rarely differentiated and widespread confusion exists about the argument that buildings can be fractal (Ostwald 2013, p.648).

Then what buildings or architectural designs are in reality? Are they fractals or not? Ostwald (2001; 2003) notes that,

> From a mathematical perspective, buildings may have fractal dimensions, but they are not examples of fractal geometry. Moreover, as Stanley and Meakin (1988) suggests, buildings are actually part of a general class of objects called multi-fractals, a class which covers most natural and synthetic objects in the material world (ibid, p. 648).

Only in mathematical images and to a certain level in nature, ideal fractals are found. Ostwald (2001; 2003) posits buildings and cities are also multi-fractals; every building has several levels of dimensionality, ranging from the cellular, granular, and material to the textural, constructional, and formal (ibid, p. 648). Yet these several levels of dimensionality or scale is important for an inhabitant as these dictates the living experience in the space. However, Paver Tucker and Jasnoska (2013, p. 94) critically scrutinize the fractal dimension as follows,

> Based on this observation, Myint warns that the usage of fractal geometry for description of cities as a whole may lead to considerable confusion [14]. Another Problem emerges from the calculation of fractal dimension. Since real-world objects cannot be described in a mathematical way, there is no exact formula for calculating the fractal dimension.

The argument made is partly true about considering cities as fractal without a sufficiently improved method of analysis and right interpretations of the data obtained, however deriving a real world phenomena in patterns (Alexander's perspective) is not impossible. Fractals of nature are true features, not a falsified opinion. Our cities, houses can not be detached from nature. In this line, Myint could be right about the calculation of fractal dimension appropriately. Though, extensive improvements have been done until the year 2016 i.e Ostwald and Vaughan's book *Fractal Dimension of Architecture*, the question remains when we read a dimension of the 3Dimensional building in 2D drawings or photographs, how much it deciphers? Does it tell the whole story? In the midst of these arguments, the comprehension of multi-fractality requires multi-dimensional study of buildings to appropriately understand built environment. How can it be made possible? The research question was shaped by the following two arguments. Bovill (1996)'s hypothesis "as it is possible to measure the fractal dimension of a site or environment, and then generate a design with the same fractal dimension, to produce a visually coherent addition to a location" (Ostwald and Vaughan, 2016, p. 34). It seems undeniably true. This very hypothesis was used by Jon Cooper (2005) using two 'r' words; respect and reflect, where a fractal character (dimension) of existing street edges was measured and used to develop a new architecture to address two 'r'. This case is explained in detail in Chapter 4. The second argument is by Pearson (2001) who critically notes how fractal geometry is applied only externally which has completely separated from interior functions of the building, is the key to understand limitations of fractal architecture measuring techniques (Ostwald & Vaughan 2016, p.25). The present methodology of finding fractal dimension rests on the planar basis, either photographs or drawings. It turns the whole into pieces and the pieces only represent the whole in limited sense whereas architecture is merely about space, the whole i.e 3D.

Another thought provoking argument of inherent fractal understanding in human comes from psychological evolutionary perspective. Joye (2007) notes the attraction towards natural contents and to particular landscape configurations which affect human functioning and help to reduce stress. This notion relies on how human evolution took place in natural settings like forest, caves, open plain terrains, under the sky and so on. All of which are examples of fractal geometry. Moreover, Joye argues, how evolution has „...subtle but non trivial adverse effects on psychological and physiological well-being" (2007b; p. 305). The existence of cognitive modules that possess peculiar information that process perception and conception of the natural setting or objects are argued by cognitive psychologists. Mithen (1996) and Pinker (1994) argues, as cited by Joye (2007b, p. 306), such modules have been evolved along with the survival chances and challenges, for example, finding food, protection from wild animals and so on. Based on

Parson's (1991), as cited by Joye (2007, p. 318), argument on the involvement of different visual structures in modulating stress hormones describes how different settings creates an autonomic stress response. This is an essential phenomena to understand the visual perception of settings or objects which reduces or increases the stress in the observer. In the same line, Prof. Richard Taylor's extensive research on Jackson Pollock's drip paintings, its fractal analysis and ultimately the answer to the question why fractals are soothing (Taylor 2017/ The Atlantic[4]) show interesting findings. On fractal analysis of Pollock's unusual method of making artworks and how they represent the fractal quality, Taylor (2017) argues, on comparing the Pollock patterns with the forest, both of them show exactly the same fractal qualities. In an experiment of participants' stress level test, low to mid range fractal values between 1.3 to 1.5 of computer generated fractals and of natural vistas with similar fractal dimension range are shown to the participants and their brainwaves are measured using EEG. During the experiment, „ ...the subjects' the frontal lobes easily produced the feel-good alpha brainwaves of a wakefully relaxed state" (Williams, 2017 *The Atlantic*; p. 4)[5]. The research finding also supports the Biophilic (Biophilia mean love for life) nature of fractals as supported by Joye and other environment psychologists. Based on Ulrich's (1983) psycho evolutionary framework, Joye (2007b) plausibly notes Kaplan's (R. Kaplan & Kaplan, 1988; S. Kaplan, 1989) preference Matrix of four structural landscape properties namely complexity, mystery, coherence & legibility to have a positive aesthetic evaluation and positive influence. This preference matrix is an essential tool for designers and architects to exhibit these qualities in their design which can be executed using fractal dimension analysis.

Though in complete contradiction to Mandelbrot's observation, Taylor's (2006) argues on Frank Gehry's deconstructivist design as organic and stress reducing fractal architecture in line with

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5 Williams, Florence (2017 Jan. 26) Why Fractals Are So Soothing?, Article in The Atlantic. [online] Available from: https://www.theatlantic.com/science/archive/2017/01/why-fractals-are-so-soothing/514520/ [Accessed: 02 May 2017].

Jackson Pollock's art-works (p.239) whereas Mandelbrot doesn't consider Gehry's designs to be fractal rather repetitive (Ostwald & Vaughan 2016, p.35). This again reveals the discrepancy in propagating the notion of fractals in architecture. However, these claims are solely based on the appearance of a design in 2-dimensional planes where the concept of space has not even entered. On the other hand, in agreement with Salingaros (2004), Joye scrutinizes the postmodern and deconstructivist architecture claiming these designs are not fractal and are in the deliberate destruction of the 'contextual fit' or coherence (Joye, 2007 p. 311). Trivedi's (1989, p.249) study on Indian temples, as cited by Ostwald & Vaughan (2016; 26) shows that the actual fractal nature inherited in these temple designs by arguing, "a building type that features both recursive and rule-based geometries that conform more closely to the expectations of fractal geometry." Ostwald & Tucker (2007) mentions the attempt by Oku (1990) and Cooper (2003; 2005);

> ...to provide a quantitative measure of the visual qualities of an urban skyline, Yamagishi, Uchida and Kuga (1988) have sought geometric complexity in street vistas and Kakei and Mizuno (1990) have applied fractal geometry to the analysis of historic street plans; a project that has been extended by Rodin and Rodina (2000).

In realizing that architecture is not only about small scales rather it exists in every scale i.e. from small to larger scale. That is why it is important to understand how fractal analysis is understood in a larger scale. For the same, Ostwald & Tucker (2007 argue,

> At a larger scale Cartwright (1991) offered an overview of the importance of fractal geometry in town planning and Batty and Longley (1994) and Hillier (1996) have each developed increasingly refined methods for using fractal geometry to understand the visual and growth patterns of macro-scale urban environments.

However one thing is certain, these contradiction interpreting the fractal analysis and its validity calls into the question of how fractal understanding is measured just beyond the definitions of words.

To get an overview of the present day researcher's opinion on the issue. My personal email correspondence with several researchers in the field explains little more on the issue. Wolfgang Lorenz (2017 May 20) gives a two-fold explanation of fractal dimension: the first one deals with the measurement of fractal dimension of any facades or plans whereas the second deals with the accurate interpretation of the data obtained. Although both of the issues are being addressed by current researches only, there is not much revealed in the part of the interpretation. Lorenz (2016) argues the use of fractal dimension studies revealing the nature of modern architecture which is often considered as ‚scalebound' objects. The ‚scalebound' object shows a limited number of distinct visual elements in several scales which can be studied using box counting method.

Fractal dimension measurement is a mathematical tool which depends on various ingredients from the picture or facade drawing used for analysis till accuracy of the software. It is essential to minimize the influences that are observed while analyzing the facades using photographs or elevation drawings. A well-defined system of frameworks and refining method is explained in *The Fractal dimension of Architecture* (Ostwald & Vaughan 2016) which is explained in Chapter 4. The interpretation and meaning of the result are used to characterize past design schemes and their visual complexity. It can be a helpful tool to devise classifications. Moreover, the transformation of existing fractal dimension character of a neighborhood into new designs is an essential tool for regulations of some specific zone with specific characteristics. Professor Michael Ostwald (2017 May 31) calls fractal dimension, an abstract concept which deals with „distribution of information in a set of data". In case of architectural drawing or photographs, this information refers to geometry associated with design. Any two buildings can have same fractal dimension (2D) yet with different geometric forms. However, in agreement with Lorenz, Ostwald claims certain hypothesis and arguments can be tested with fractal dimensions. These two observations and explanations on fractal dimension remain important for the research question. 2D figures are influenced by several factors like what is in the picture and how it has been analyzed using software whereas space is an entity with the third dimension which enables one not to be dodged with overlapping lines or geometries seen in 2D figure though being well apart in space. Regarding the same issue, Professor Jon Cooper (2017 March 31) explains our false understanding of buildings as a solid cube or Euclidean geometry rather a building is full of holes.

The whole space is rough and this roughness or detailings can be measured in several scales which fractal dimension provides. In particular fractal dimension of space is vital to understand real design intention in this case. This comprehension is the boon for the Un-Folding space method where the very space is considered as a hole in 3D world similar to a hole in a flat plane.

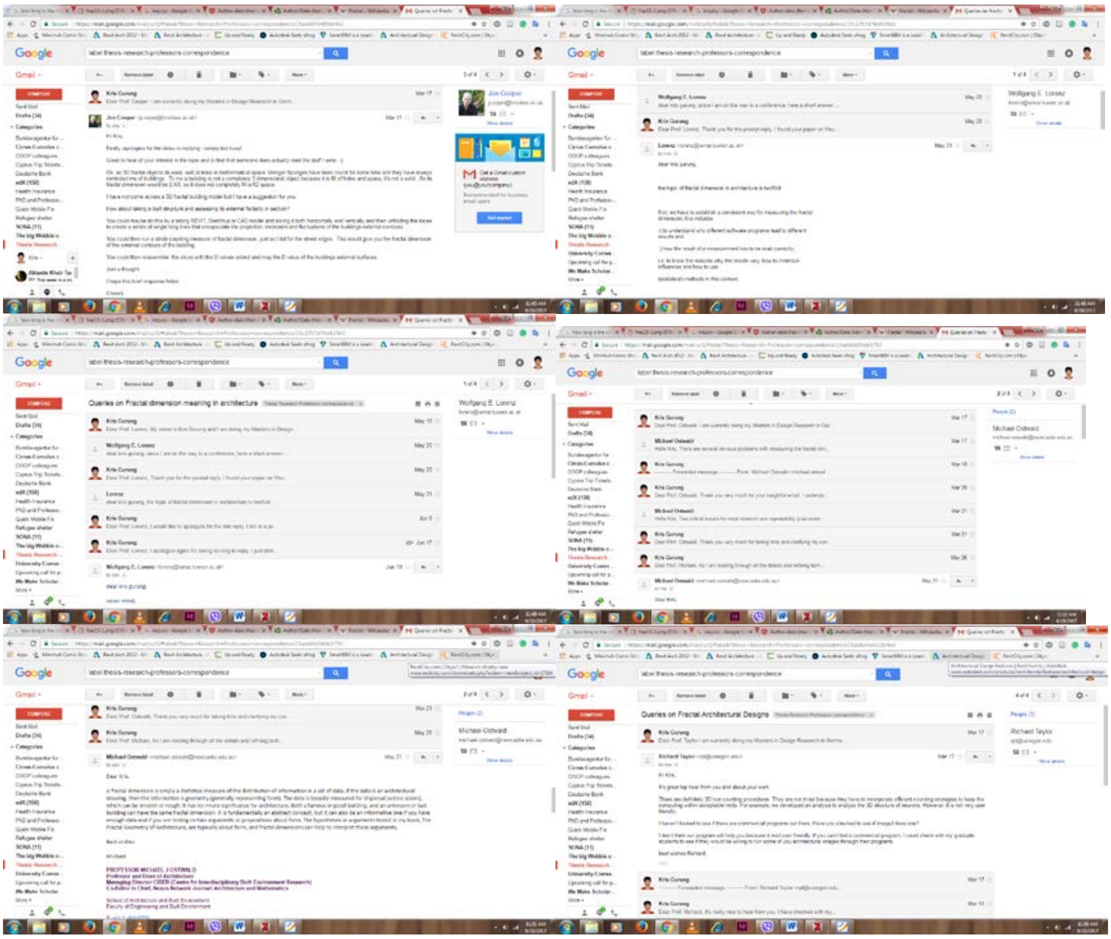

Fig. 12 few screenshots of investigative email correspondence with the professors in the field of fractal dimension studies

3.1. Historical Account: Fractals in Architecture

After the publication of *The Fractal Geometry of Nature* in 1982, the fractal thinking began in architectural world. Initial remarks in the connection between fractals and architecture can be found in Mandelbrot's (1982; 23) argument where he praises the self-similarity exhibiting nature of Grand Master paintings or Beaux arts elevating the knowledge of the masters describing the work, an effort to imitate Nature while giving sharp criticism to Mies Van der Rohe's architecture as, "… notably in the context of architecture: A Mies van der Rohe building is a scalebound throwback to Euclid, while a high period Beaux Arts building is rich in fractal aspects" (1982; 24). However, Ostwald & Moore (1996b), as cited by Ostwald & Vaughan (2016; p.26) argues, on the contrary researches have shown that Mies's *Seagram building* possesses at least 12 scales of conscious self-similarity (Ostwald & Moore 1996; 26). It shows an indication of a subjective definition of fractals rather than objective one or one can argue the scenario being the preference of classical architecture over modern. That is why it is time to establish one concrete opinion on what is fractal and what is not? or how much fractal dimension is applicable in architecture? This dissertation is an attempt in the direction.

On a historical basis, a highly contested and intricate relationship still exists between sciences of complexity including fractals and architecture, Ostwald (2001) observes the two decades of shifting and changing common point of connection. Following two subchapters is an account of initial attempts of fractal thinking including sciences of complexity in architecture.

3.1.1. Rise of Fractal Architecture

Following three case studies, House IIa, Moving Arrows, Eros and Other Errors or the Romeo and Juliet Project and Extension of Victoria and Albert Museum are pioneering attempts that used fractal theories in architecture.

House IIa

After Mandelbrot's first English edition *Fractals: Form, Chance, and Dimension*, in 1977, *House IIa* of Peter Eisenman is considered as a conscious attempt of using fractals in architectural designs and the philosophy was based on complexity sciences as this became "a central thematic motif in Eisenman's housing design produced during the Cannaregio design seminar in Venice " (Ostwald 2001) in 1978 July. The concept of scaling was illustrated by Eisenman as, "…a process philosophically as entailing [']three destabilizing concepts: discontinuity, which confronts the metaphysics of presence; recursivity, which confronts origin; and self-similarity, which confronts representation and the aesthetic object['']"(Ostwald, 2001). To acquire multiple or infinite scales of self-similarity, several objects, scaled version of *House IIa,* were placed over the whole area of the Town,

> ... the smallest object being man height but obviously not a house, the largest object plainly too large to be a house, and the house sized object paradoxically filled with an infinite series of scaled versions of itself rendering it unusable for a house. The presence of the object within the object memorializes the original form and thus its place transcends the role of a model and becomes a component and moreover a self-similar and self-referential architectonic component (Ostwald, 2001).

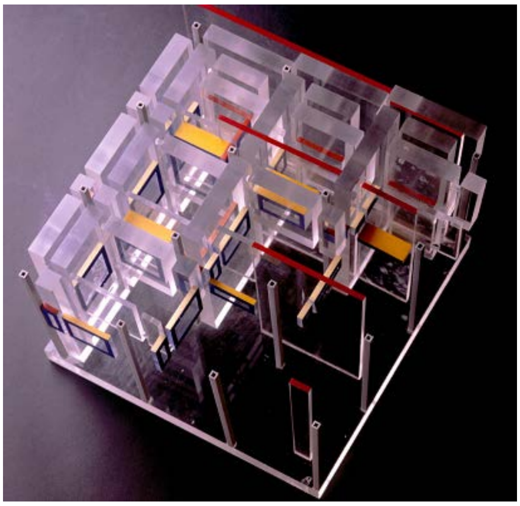

Fig. 13 House IIa Model

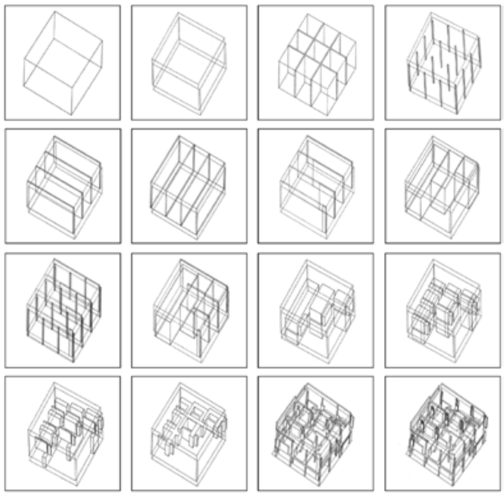

Fig. 14 Right: Analytic Diagrams

The attempt was highly appreciated considering it as a new approach for underpinning philosophical foundations in the architecture fraternity.

Moving Arrows, Eros and Other Errors or the Romeo and Juliet Project

Moving Arrows, Eros and Other Errors or the Romeo and Juliet Project is Eisenman's 1985 project that extrapolates the generative concept of scaling into architecture. Though project received many critical acclaims, critics such as Aron Betsky argues, as cited by Ostwald (2001) the concept of 'normal scale' not fitting in Mandelbrot's scaling;

> [u]sing a formula developed by the scientist Benoit Mandelbrot, which determines the 'self-sameness' or autonomous replication inherent in certain figures, [Eisenman] mapped plans of vast territories over each other. This technique questioned architecture's relation to a 'normal scale' and 'problematized' the concept of human perspective.

The concept of human perspective is anthropocentric which shows the parochial attitude of human civilization. Though the history has been guided by this notion with some exceptions like those illustrated in *African Fractals* by Ron Eglash (1999), Eisenman argues,

> [f]or five centuries the human body's proportions have been a datum for architecture. But due to developments and changes in modern technology, philosophy, and psychoanalysis, the grand abstraction of man as the measure of all things, as an originary presence, can no longer be sustained, even as it persists in the architecture of today. In order to effect a response in architecture to these cultural changes, this project employs another discourse, founded in a process calling scaling (Ostwald, 2001).

Eisenman's argument tries to deny the long lived anthropocentric perspectives on architecture which are heavily guided by literary veterans like Da Porto, Shakespeare. The conventional paradigm in architecture is jarred and shifted with this idea of convergence of ‚reality' and concept (Ostwald 2001).

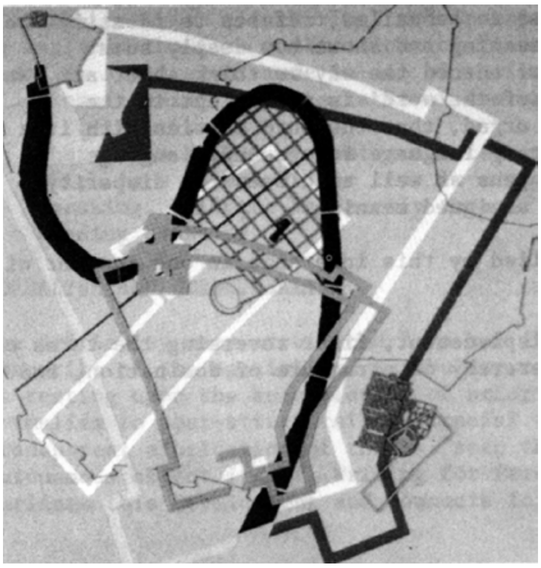

Fig. 15 Master plan sketch

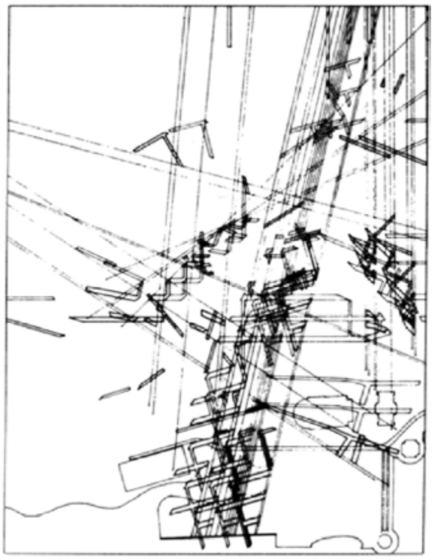

Fig. 16 Sketches

Extension of Victoria and Albert Museum

Daniel Libeskind's projects are also equally opposing the linearity and determinism in architecture. Libeskind designed several projects addressing complex phenomena and theory of complexity. One of them is extension of Victoria and Albert Museum in London where chaotic spirals rendering columnless form was designed. The design reconsiders the growth phenomena which bears complexity in its generation and being into a form.

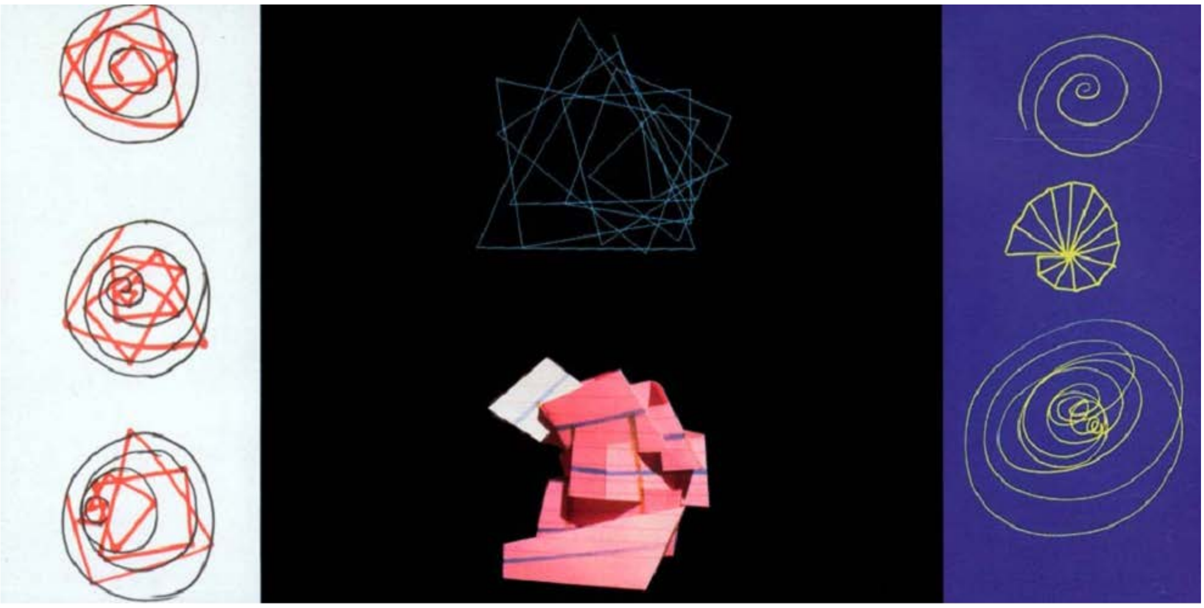

Fig. 17 Cecil Balmond: Victoria and Albert Museum Extension conceptual model (source: Herr, 2002)

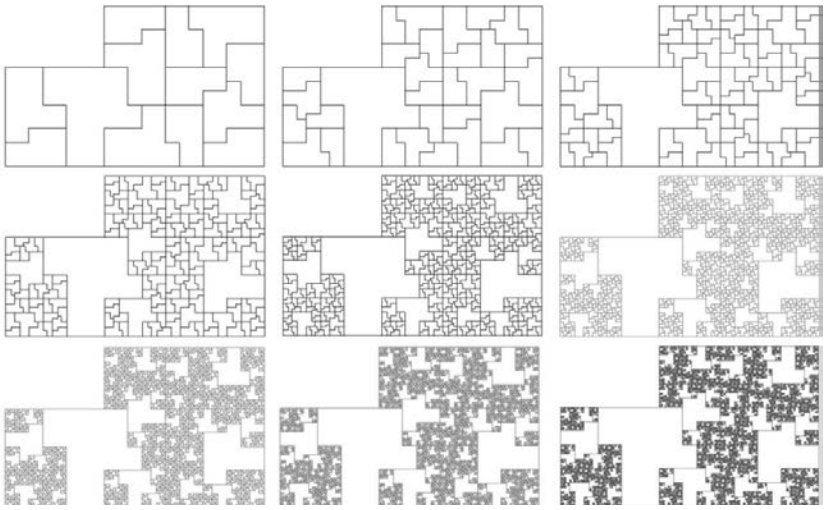

Fig. 18 Generative process of design

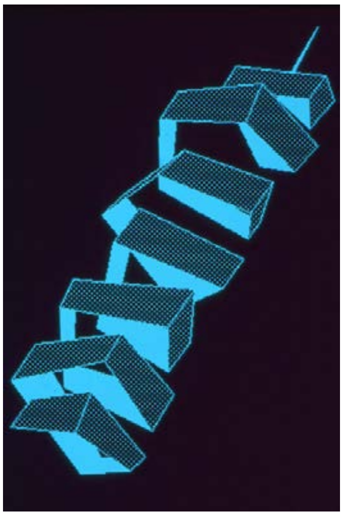

Fig. 19 Columnless structure

3.1.2 Skepticism in Architecture community

With an equal critical acclaim, the skepticism and criticisms on fractal thinking in architecture were observed after the late 1980s. Michael Sorkin's critics on Coop Himmelblau's surrealist concepts like "automatism" with the rhetoric of complexity science including "interference", "chaos", "indeterminacy", "iteration" and "open systems" (Ostwald & Chapman, 2009) is noteworthy criticism in Architecture. *In Post Rock Propter Rock: A short History of Coop*

Himmelblau, Sorkin declares that "[c]haos may be a little over familiar nowadays, especially in its studied inscription in architecture. However, the idea behind this latest upheaval in physics does have real implications for us" (Ostwald, 2001). Though Sorkin accepts the importance and applicability of chaos and fractals in physical and biological sciences, its use in architecture does not seem appropriate. In 1990 Aron Betsky, as cited by Ostwald (2001), described Eisenman's *Biocentre* at the J.W. Goethe University of Frankfurt, a geometry corrupted by a parasitic mesh of Fractal geometry. Betsky questions the legibility of fractal geometry, its importance rather one can say its negative effect like a parasite corrupting the Euclidean geometry. This observation/criticism also sheds light on the state of ill-definition of fractal geometry's relation with architecture i.e. why and how fractal geometry is important in architecture? has not been answered or justified. In the same line Gisue Hariri and Mojgan Hariri in 1993 posit in their manifesto for architecture that they do not follow 'Trends' and despise 'Kitsch' like 'Chaos' calling the science of complexity a mere Trend (Ostwald, 2001). Moreover, British architect and critic (1994) describes the term "a furor of nonconsensus" in architectural theory by providing a description of un-named architectural role models in a synthesis of Peter Eisenman, Daniel Libeskind and Morphosis as cited by (Ostwald, 2001);

> Here is a man who scatters chaos on paper and talks about randomness and fractional theory. He calls the scatter the plan of a building. Anything will do-twigs purloined from a pigeon's nest, notes transcribed from the Song of Songs—a scribble he did with his eyes shut, like a shaman in a trance drawing in the dust of the Nevada desert. His building is built. It appears like a mirage in the wasteland of the city, a histrionic essay of joints and materials. He claims the building is ambiguous-he says it is like the chaos of modern life-he tells us all that it is profound.

The argument is a well-observed one, however, chaos is true, complexity in systems like architecture is true and fractality of nature is true. Thus this leads scholars to better equip themselves to understand chaos and fractals rather than in philosophies and metaphors. A mere requirement that could give a consensus of fractal understanding.

3.2. Pattern language and Fractals: An overview

Christopher Alexander (1977), the pioneer of Pattern language in architecture, emphasized the need for understanding patterns in relation to functional needs and environmental response. *A Pattern Language* explains 253 patterns or units which together form a design statement or a design language, the language of architectural space. One can make an analogy between a language (spoken or written) where words are put together to form sentences. If words are put together well (grammatically) then the sentence is formed well and if the opposite happens the sentence is not understood. So what is a pattern? It is a socio-geometric solution, as Salingaros (Sustansis 2015 July) explains, a geometric solution, say forms/shapes to a socio-cultural life. For instance, in a house, circulation may be required to go through the main living room and the main living room is a social space for the family which needs to be well lit and ventilated and from where more private spaces may be connected. These patterns for different spaces connect with each other to give a connection. These connections of different patterns can be expressed in thousand different geometric forms. There are many possible ways to connect these words to form a sentence which has a meaning. Thus it is not restricting design thinking rather providing a way to create meaning with patterns. It also addresses the need for different pattern language for different regions which addresses the vernacular architectural practices which resembles with different language in different regions.

Fractals, on the other hand, shows patterns existing in nature. It represents how a whole form is coded in one simple rule or pattern. Salingaros (2003), one of the key figure in new urbanist movement, argues the need, essential qualities for a city to be alive, is inherent in nature; in its fractals. The well-connected cities with several layers of connectivity from pedestrians, vehicular movement up to high-tech modernist car connections make the city alive triggering each node of the urban fabric to be active. A code or pattern that generates the whole urban fabric to be a mixed use living city which shows scaling and connectivity at every level of the fabric. The prime rule of modern urban planning, monofunctional zoning system divides the fabric into different zones with strict boundaries such as residential zone, industrial zone, commercial zone etc so as to function according to the name. In contradiction, this system renders zones dead in different times of the day. For instance, as Faegre (2009) argues, the residential zone is dead during the day time when working people commute to work in the commercial or industrial zone whereas these commercial and industrial zones become dead during the night when people return to their residential zone. In conclusion, no two zones are alive at the same time. The reason for such an outcome is linear Newtonian thinking whereas it needs a fractal thinking.So what is fractal thinking? Jane Jacobs provides a simple description as cited by Faegre (2009; p.2);

> [Fractals are] complicated-looking patterns that are actually made up of the same motif repeated on different scales…. For instance, a muscle is a twisted bundle of fibers. Dissect out any of those fiber bundles, and you find that it, too, is a twisted bundle of fibers. And so on….That's a real-life fractal.Mathematicians make computer-generated fractals, fascinating in the complexity and seeming variety, yet each fractal is made of repetitions." [p.22, The Nature of Economics, Jane Jacobs].

In the case of land use planning, it is essential to interpret the fractal thinking in a way where mix land use plan is proposed and expanded according to the need of urban fabric. It is essential to understand that the concept of strictly separate zoning for example commercial land use can not be incorporated or completely incompatible in residential or industrial land use is completely aligned with linear thinking which does not resolve the problems. Rather a pattern of incorporating all necessary land use in a sizable framework and analysis of the effect and living characteristic of such land parcel is important. Then expanding such an effective pattern using fractal growth of land parcels in the urban fabric ultimately generates a living city.

3.3. Topology and Architecture

Topology is a new way of understanding space in mathematics where Topos means the place or space (Kantor 2005). It is also popularly known as the geometry of position where two objects or forms are considered as one if one can be transformed into another with continuous deformation without doing any cuts or leaps (Kantor 2005). In ancient Greece, *Stigma* (a puncture) and *Semion* (a sign) describe a point where the first one defines a space itself and the latter in the space of something else. The important point in this description is geometry begins with a point (ibid p. 13). Then, from point to a line, plane and volume are formed with subsequent movements in the previous geometric figure i.e. a line is created when point starts moving in a straight path and so on. Initial attempts, after understanding the limited knowledge in defining forms of geometry, were carried out by Gottfried Wilhelm Leibnitz in the 17th century in his book „Characteristica Geometria". As cited by Kantor (ibid; p.13) in 1679 Leibnitz writes to Huygens: "We need another strictly geometrical analysis which can directly express situm in the way algebra expresses the Latin magnitudem". After Leibnitz, several other mathematicians start analyzing the newly coined term ‚analysis situs' which means analysis of position. Yet the story behind the invention of ‚geometry of position' is an interesting mathematical problem solved by Euler in 18th century. Euler (1735), as cited by Kantor (ibid: p.14), on resolving a hypothetical problem regarding crossing a bridge of a city of Koenisberg invented the new nature of geometry namely ‚geometry of position' which means „determining position and for seeking the properties which result from this position, without regards to the sizes themselves" [Euler 1741].

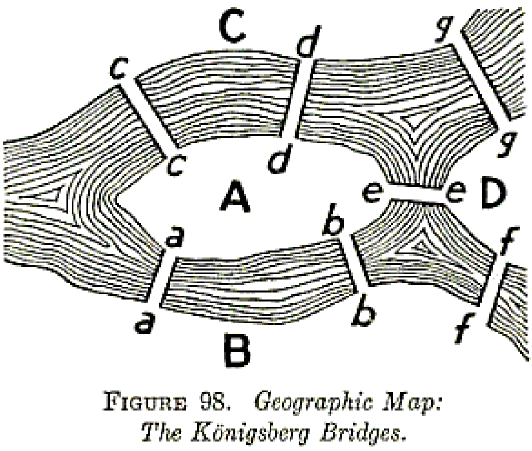

Fig. 20 Graphic representation of Koenigsberg. A, B, C and D represents land parcels whereas a-a, b-b, c-c, d-d, e-e, f-f and g-g represent the seven bridges connecting the land parcels.

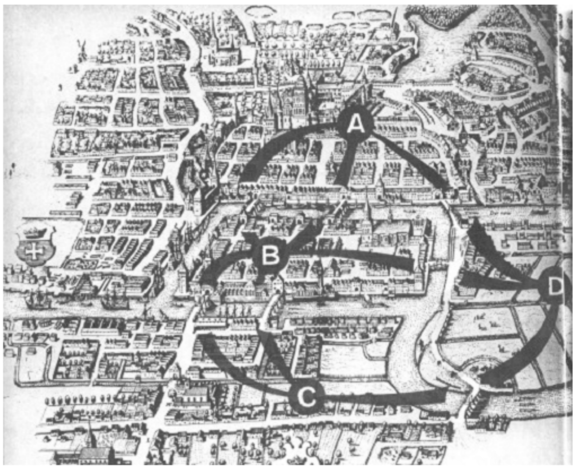

Fig. 21 An actual map of the Koenigsberg city (Kantor 2005)

A case of Koenisberg where an island surrounded by a river is split into two branches with seven bridges connecting different land parcels. A problem was reported asking if it is possible for a person to traverse all the bridges in a single trip without doubling back as well as ending the trip at the same place where it began (Wolfram mathworld).[6] Euler provided a general solution with a negative answer which nullifies the geometrical characteristics; distances, length of bridges, angles and so on, of the problem. It is also considered as the beginning of graph theory when he developed the second sketch. It represented the problem as a problem of position and properties associated with it regardless of the sizes. These sketches are considered as the first manifestations of topology that deduce the geometric problem into its essence i.e a more flexible structure of topology (Kantor 2005).

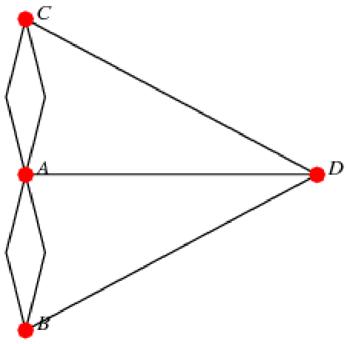

Fig. 22 problem reduced into lines and points

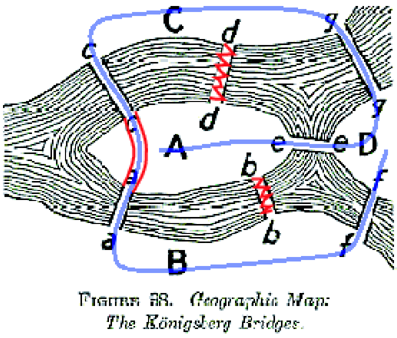

Fig. 23 Figurative presentation of the possible traverse

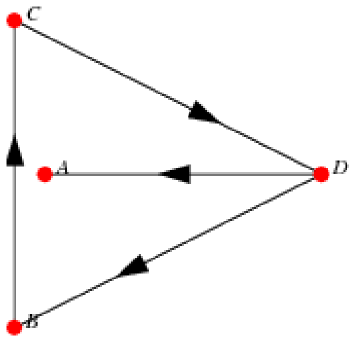

Fig. 24 Graphical representation of the traverse in lines and points

The land parcel or bank, C is denoted by a point C which provides two bridges to reach the island A. Similarly land parcel C and D are also denoted by points C and D respectively in Fig 19. whereas the connection among these points is denoted by lines joining the respective points and the bridge by line segments. The graphical representation of the actual geometric figures like the rivers and lengths are not taken into account or not considered.

The Fig. 19, Right image represent the actual traverse proposed by Euler concluding the problem

6 http://mathworld.wolfram.com/KoenigsbergBridgeProblem.html

is not resolved in only one single traverse (Wolfram mathworld)[7]. However, this provided mathematicians a new sense of freedom where one can twist and deform the spaces and real world problems without changing the topological structure and actual result (Kantor 2005). This gave a way of thinking where a donut and a coffee cup with one handle is considered same. It means one can twist and deform a donut constantly to shape it into a coffee cup and vice versa. That is why topology is considered as the mathematics of rubber. On the other hand, architecture, innately about forms and spaces, can be visualized in terms of this flexible presentation of geometry and space. Moreover, the graph theory initiated by Euler is an important aspect for architects to resolve complex spaces into graphical images of points, lines, and other shapes.

7 http://mathworld.wolfram.com/KoenigsbergBridgeProblem.html

4 MEASURING ARCHITECTURE

4.1. Architecture in terms of Dimensional Analysis

The form defines how an object appears in view. However, the untouched part of the form is how it tells the design idea, underlying philosophy and other characters intended by the designer. What a form of an object is; its 'Geometry'. The geometry reveals the physical characteristics of the object, moreover, it certainly delineates the perception of the viewer i.e how an object is being perceived. In understanding, this perception mankind started using dimensions intuitively. Basic examples of dimensioning can be found in measurement systems like length (from hand measurement to tape measurement and so on). Ostwald and Vaughan (2016; p.7) states "[The] dimension is physically tangible (it can be touched and otherwise sensed) and it has a practical material and scale limits, meaning it cannot be infinitely divided or enlarged." This argument clarifies the way a dimension is understood, it can be touched, explained, for instance, a table can be described according to its length, breadth and height (which are dimensions in three different axes), mass, volume, specific gravity, material and so on. On the hind sight, the argument posits on sensing the dimension which makes dimensional analysis more interesting. Something that can not be touched, can be sensed and explained accordingly. The feel and warmth of the material, the texture or roughness or its surface of a table are examples of another kind of dimension which is more descriptive than exact. It makes this dimension to be experience based and intuitive. If this tangibility and experience aspect of dimension is well defined then it can dictate the possibility of clearer understanding of architectural spaces. So how do we analyze the world around us using the dimension? How do we build up analogies or relations among different aspects of the forms being analyzed? For the analysis of such dimensions, Bridgman (1969) explains "The principal use of dimensional analysis is to deduce from a study of the dimensions of the variables in any physical system certain limitations on the form of any possible relationship between those variables" (Sonin, 2001; p. 6). This analysis critically argues on behalf of perception of things from its dimensional characteristics. Moreover, it is similar to the graph theory discussed in topology and architecture sub-chapter, where a real world problem is reduced into simple forms and shapes to understand. Dimensional analysis reduces the whole (eg. Market rate, Hospital project in a rural area, new education method etc), a complex system, into its several parts (a part could also possess a complex structure possessing many components within) and the relationship between these components is carried out. One interesting aspect of our world is that it has been analyzed and debated from many different perspectives to better understand its multidimensionality. For mathematicians and scientists it is called 'Euclidean space' whereas for philosophers it is 'material world' and for architectural theorists, it is known as 'lived space' or 'experiential space' (Ostwald & Vaughan 2016; 7). So for the purpose of this dissertation, the perspective of lived space or experiential space is taken to further analyze the main research question.

In case of architecture, the space enclosed by walls and roofs opened outside through doors and windows, the skylight is an interesting and equally important topic to explain in detail. This very space has been described by many architects and designers in terms of metaphors, symbolism, spirituality, silence and so on whereas on the other side of the coin, building science (merely physics of building functioning) carries out experiments to collect data and represent them as statistical proofs for comfortable indoor climate based on temperature, air movement, thermal mass of walls, materials and so on. This dissertation also tries to explain the lived space with dimensional analysis taking fractal dimensions.

The notion of non-integer dimension is clearly explained in the definition of fractals by Mandelbrot "…a set for which the Hausdorff Besicovitch dimension strictly exceeds the topological dimension" (1982; 15). So if fractal analysis of a building is carried out then what a fractal dimension exactly measures? It, in simple words, measures the roughness of surfaces being analyzed. If we take a building, for example, a building is made out of surfaces, these surfaces are not smooth rather rough, these surfaces make the space, ultimately space is not smooth, it is rough. This roughness is observed on many scales. The roughness of plans, facades and of the space is measured using fractal analysis. Fractal dimension analysis exposes several layers of information hidden in the architectural project which none of the other models can provide i.e. visual complexity over multiple scales. The term refers to the recursion of similar features over multiple scales or information (geometric shapes like lines) distribution over several scales. What is the importance of studying this visual complexity in several scales in architecture? The concept of scale is very essential, especially human scale. As Christopher Alexander in his book *Nature of order* (2002) points out 15 properties of architecture which give rise to 'phenomenon of life', the scale, particularly ranging from 2m to 1cm which signifies human scale, marks a significant characteristic of any design; designed for humans. The details in this range shape our space, perception of things one sees and thus makes the environment livable. The fractal dimension which tries to find out details in many scales corresponds to the logic.

Architecture has its roots extended to many interpretations for better comprehension in different dimensions. For some, it's pure geometry, for some relationships between what is built for who. From a building's physical appearance to its intangible and abstract dimension, architecture is analyzed. Varodius & Psarra (2014; 91) argues in the same line about architecture being, "[...] spatial relations that accommodate functions, afford social relations and create visual interest. Through openings and walls, architects manipulate continuities and discontinuities of visual fields in two and three dimensions". This continuities and discontinuities of visual field correlate with the studies of fractal dimensions which elucidates the visual interest mentioned by Varodius & Psarra. In the space syntax school, as Varoudis & Psarra (2014; 91) argues, researchers explore the spatial relations and visual fields rather than geometrical objects which signifies the importance of understanding space in terms of dimensions of visual field rather than strict geometry. Therefore the dimensional analysis from different approaches like fractal analysis is important as well as fruitful to gain more knowledge about the space we live in.

4.2. Dimension: A notion

Fractals, in a way, is a rebellion against calculus since calculus thinks surfaces to be smooth which are overly idealized, on the other hand, fractals consider them to be rough. In addition, only perfectly self-similar shapes are fractals, is also an overly idealized concept which is completely against the pragmatic nature of fractals which suppose to model nature itself which is not completely self-similar. The real idea of fractal is the concept of fractal dimension. There are many ways to describe dimension in Geometry. What does dimension like 1, 2 or 3 mean? how fractal dimension came into being (mathematically)? are the questions that will be answered in this chapter. Based on self-similarity and scaling factor, a simple yet easy to understand analysis (source: 3Blue1Brown, 27 Jan.2017, Fractals are not typically self-similar/youtube) is discussed as follows:

Starting with perfectly self-similar shapes (Euclidean geometry) like a line, a square, a cube and a (Fractal geometry) Sierpinski triangle. A line can be cut down into half (scaling factor of ½) giving perfectly self-similar two lines to that of the whole line. It means when one of the two lines is scaled up by 2 then one gets the whole line again. Likewise, a square can be scaled down to four pieces by a scaling factor of ½ where every piece is exactly the same as the parent square which means on scaling one of four small squares by 2, one gets the whole square again. A similar thing happens in case of a cube and a Sierpinski triangle. Sierpinski Triangle is made out of three exactly self-similar parts. It can be scaled down by ½ giving rise to three pieces. So what is the relation between scaling factor and a dimension? Let´s establish a relationship between scaling factor and the way mass is reduced while scaling because the dimension of these shapes has everything to do with how the mass changes as one scale them:

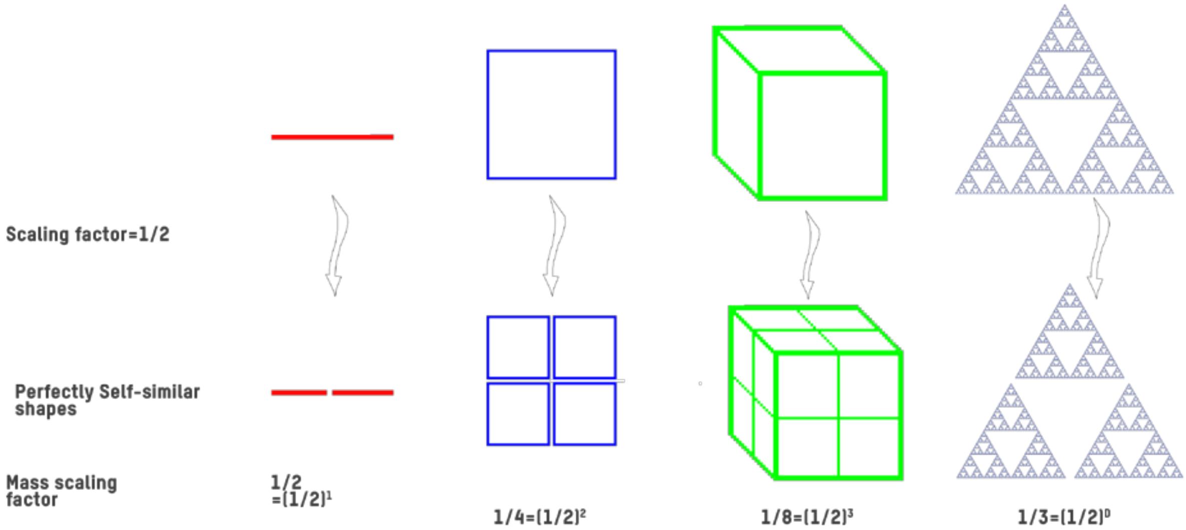

Fig. 25 A Graphic representation of relation between scaling factor and self-similarity in defining dimension of an object

1. When a line is scaled down by ½ i.e. scaling factor is $(½)^1$, the mass is scaled down by ½ which can be seen viscerally.
2. When a square is scaled down by ½, the mass is scaled down by ¼= $(1/2)^2$ as four pieces of a small square make up the whole original square.
3. When a cube is scaled down by ½, the mass is scaled down by 1/8= $(1/2)^3$.
4. When a Sierpinski triangle is scaled down by ½, it is likely to say the mass is scaled down by 1/3, but the way masses of line, square and cube are scaled down by clear integer exponent as 1, 2 and 3, however, Sierpinski Triangle doesn't show similar characteristic.

On examination, what it means for a shape to be, for example, two dimensional? It is what puts the 'two' in two dimensional. When one scales a shape (2 dimensional) by some factor, its mass is scaled by that factor raised to the 'second' power i.e. 2. This power, 2 is what the dimension of that shape is. Similarly, What it means for a shape to be, for example, three dimensional, is that when you scale it by some factor, its mass is scaled by that factor raised to the 'third' power i.e. 3. So this provides a simple concept of dimension. In analyzing Sierpinski triangle to find out its dimension, if one scales down Sierpinski triangle by a factor of ½, its mass goes down by ½ to the power of some number and whatever that number is its dimension, let's say D for now. In addition, because it is self-similar we know the mass will go down by 1/3,

So $(1/2)^D M=(1/3)M$;   where, M=Total initial mass

Or, $(1/2)^D=(1/3)$

Or, $2^D=3$

Or, $\log_2(3)=D$; Taking logarithm of both sides

Using logarithm; D=1.585 hence, Sierpinski triangle is 1.585 dimensional. This dimension is a fractal value designating the nature of the shape to be in neither one dimension nor in two dimensions rather the shape is trying to be a plane but not yet a plane. This is an important result how the holes created on the triangle made it rough and reduced its planar property and gave fractional value. The surfaces like walls, facades, and plans show similar roughness by providing other details analogous to the holes.

Natural forms are not self-similar as opposed the ideal shapes examined above. Most two dimensional shapes (disk, hexagon or boundary of a coastline) are not self-similar. In analyzing a circular disk, when a disk is scaled up by a factor of 2, its mass is scaled up by 4 (2^2) which can be comprehended how radius or diameter of the smaller disk is scaled up by twice. But it is not possible to join four pieces of small copies of disks to make the bigger disk having four times the mass of initial disk. How can one do that? How can one exactly know that the bigger disk is four times the mass of the small disk? Thus although a disk is Euclidean geometry it does not show self-similar characteristic.

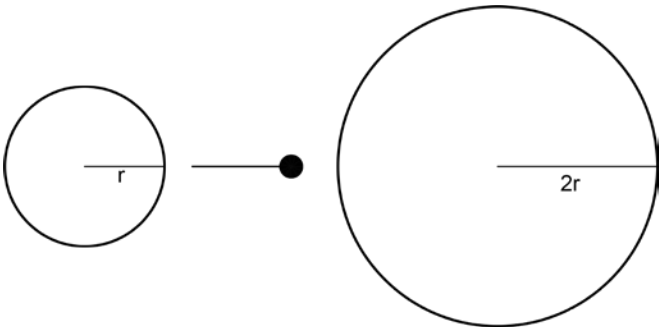

Fig. 26 A disk scaled up by 2

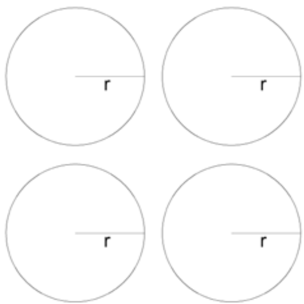

Fig. 27 four self-similar disks of radius of small disk

To correlate with the previous method of building up relationship between scaling factor and mass, here comes the mathematical solution to the problem:

1. Let's cover the disk with grid, say scale s.

2. let's count the number of boxes touched by the disk (including all inside the disk and toughed by the circumference), say N number of boxes touched. The area of the boxes together will be proportional to the area of a disk i.e πr^2.

3. Now let's scale up the disk by factor 2 and let's count the number of boxes touched by the scaled up disk. The number of boxes will be approximately 2^2N (depends on the grid).

4. Surprisingly the number of boxes has increased approximately to the proportion of 2^2 i.e. $2^2N/N=2^2$ which is squared to the scaling factor. In this case, four times as many boxes as previous one. Important aspect to know is, finer the grid, closer to the approximate value of 2^2.

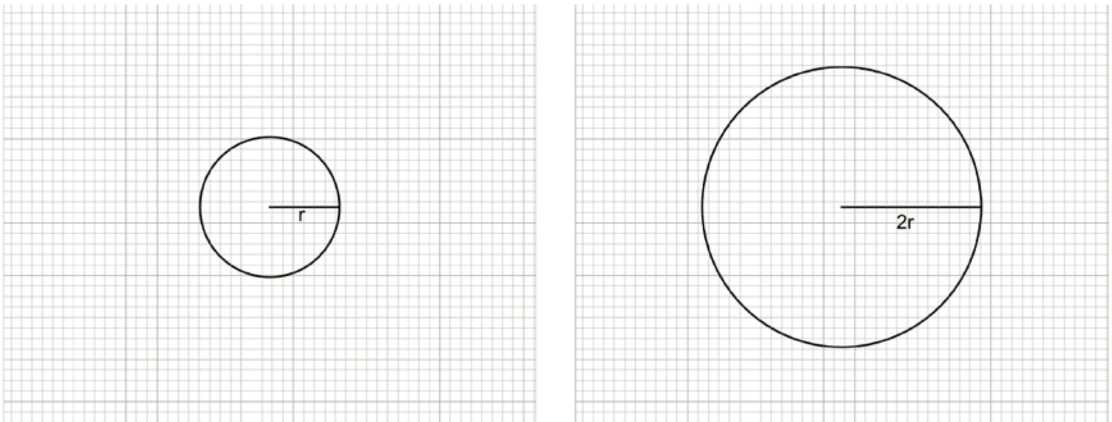

Fig. 28: Left: A disk is placed over a grid of scale S with the number of boxed touched, say N, Right: scaled up disk with number of boxes touched 2^2N

This shows a different way of finding out the dimensions of known and unknown shapes which in turn named as box counting method and the dimension (exponent of the scaling factor) is called box counting dimension. Sierpinski Triangle can also be analyzed using this method, we get the initial number of boxes be N and scaled up number of boxes be 3N (when scaling factor is 2) i.e. three self-similar shapes make up the sierpinski triangle.

As mentioned earlier in chapter 02, the coastlines are a typical examples of length variation with the scaling factor. The length measurement seems inaccurate as well as not applicable in many cases. The better way to define the coastline is to measure its roughness or fractal dimension that gives a sense of jaggedness of a coastal line.

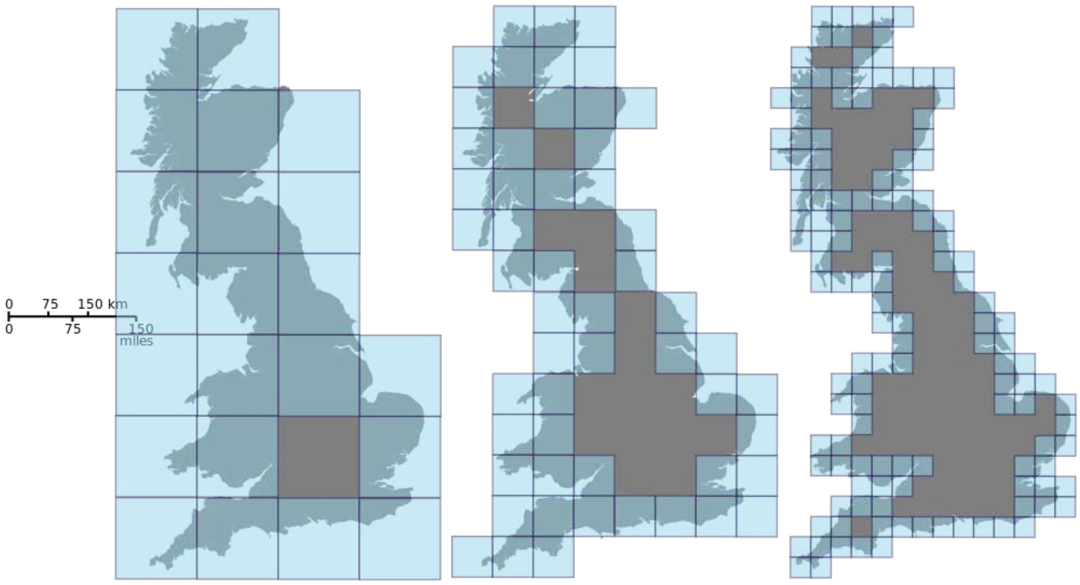

Fig. 29 Box counting Method applied to measure roughness of Coastline of Britain

With similar logic fractal dimension of any shape on a flat plane can be measured. In the figure above, scaling of box size (grid size) is used which is the reverse way to the method discussed above where shape is scaled not the grid. Map of Britain's coastline remains same but the grid over which it is placed is scaled up or down constantly for several iterations. It is important to notice, in the third picture on the right, how the boxes only touching the boundary line is counted. It is important as we are measuring the roughness of coastline. To obtain the accurate box counting dimension or roughness, the coastline map is put over several scales of the grid and the result is plotted as Scaling factor on the X-axis and Number of boxes touched on the Y-axis.

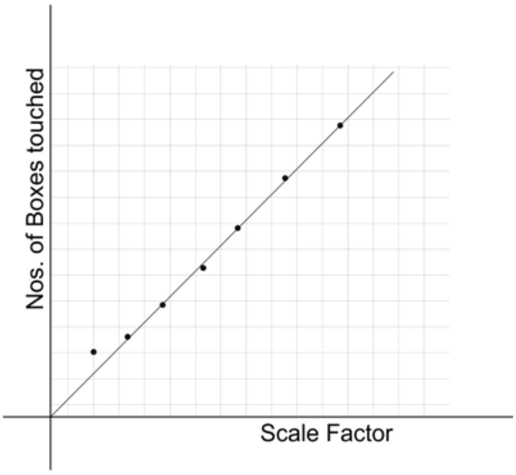

Fig. 30 Graph plot (scale vs number of boxes counted)

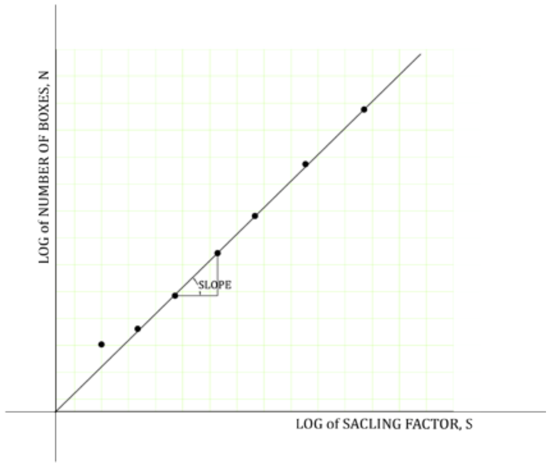

Fig. 31 Log-log graph plot indicating the fractal dimension as slope of the line

The number of boxes touched approximately increases in proportion to the scaling factor of 1.25 in case of Britain's coastline. But how to find that 1.25? From our previous analysis, a number of boxes touched, N≈(is approximately equal to) proportionality constant, C multiplied by scaling factor, S raised to the power fractional dimension, D.

or, $N \approx CS^D$, To resolve this equation one can use log of both sides,

$\log N \approx \text{Log}(CS^D)$

or $\text{Log } N = \text{Log } C + D\text{Log } S$

What this suggests is if one plots the log of scaling factor, S against the log of number of boxes touched, N touching the coastlines, the relationship will look like a line and whose slope is equal to the dimension , D.

This method, box counting method, of determining fractal dimension or roughness of surfaces is widely used in various field from physical sciences, biological science to arts and architecture and so on. Although many different ways have been invented to calculate the fractal dimension of surfaces, box counting method is widely used in architecture as this is well known, stable and mostly repeatable (Ostwald & Vaughan 2016; p.3). That is why whenever the term fractal analysis is used in architecture, it is mostly the box counting method.

4.3. Box Counting Method

Though Mandelbrot proposed seven major categories of methods for the calculation of roughness of surfaces (Ostwald, 2013, p.649). One of them is box-counting method which is our concern for this research as this method has been used to describe roughness of facades and plans of buildings. Batty and Longley (*Fractal cities*, 1994) were the first ones to use box counting methods in architectural and urban analysis, however Carl Bovill (*Fractal Geometry in Architecture and Design,* 1996) can be considered the first one to seriously analyse the architectural projects using box counting method in analyzing fractal properties of plans and elevations of several canonical buildings. Asvestas *et al.* (2000) compared all the methods and found box-counting method worked the best to find fractal dimension less than 1.8 (Ostwald 2013; 649). As cited by Ostwald & Vaughan (2016; P.12) since then it has been used for the analysis of a growing number of buildings, ranging from ancient structures to twenty first century designs (Bovill 1996; Burkle-Elizondo and Valdez-Cepeda 2001; Rian et al. 2007; Ostwald and Vaughan 2009b, 2010, 2013a).

As the basics of box counting method are discussed earlier in case of geometrical shapes and figures, it will be necessary to analyze how it has been used in architecture. Many of the scientists, as Huang *et al.* (1994, page 339) argues, call this method as naïve version since it uses basic mathematics without any refinement (Ostwald 2013; 649). It takes the orthogonal image, usually an outline, such as plan or elevation. A grid is placed over it. The numbers of boxes or cells touched or covered by the outline in the grid is counted. The second step is to reduce the grid size and count the boxes again and the process is repeated at many scales of the grid. A comparison is made between the first grid (number of boxes counted) and the second grid (number of boxes counted). By plotting a log-log diagram for each grid size, the slope of the line produced is measured which is a box-counting dimension (D_b). When the process is repeated a sufficient number of time, the data is graphed and the average slope of the resultant line is the estimated fractal dimension (ibid, p.650).

$$D_b = \frac{\ln N_{s_2} - \ln N_{s_1}}{\ln(1/s_2) - \ln(1/s_1)},$$

$N_{s\#}$ is the number of boxes in grid number # containing some details,
$1/S_{\#}$ is the number of boxes in grid number # at the base of the grid

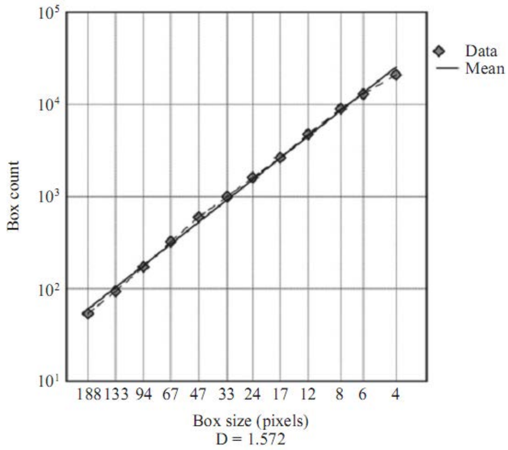

Fig. 32 Formula for Calculating Fractal dimension and the log-log plot

The following example is a fractal analysis of Le-Corbusier's famous design, *Villa Savoye,* carried out by Chalup, Ostwald, and Vaughan in 2011. This typical example shows how box counting method has been used by architects. On the left figure, one of the four elevations is put in a rectangle with only one box which is the starting point. On the second image, within the same rectangle, the number of boxes are increased with a certain scaling factor which is a crucial point in the procedure. Then the number of boxes touching the lines representing the building facades (this includes outline, windows, doors, and detailings seen on the facade). In the image, these boxes are shown in hatched type. On the third image, grid or number of boxes is increased as per the initial scaling factor that is why one can see a numerous number of boxes. As in the previous scale, the boxes touching the details represented by the lines of the facade are shown in hatched fashion. This process is carried out for 8-11 consecutive scales and the relation between number of boxes touched against the scaling factor is carried out as explained above. The figure on the right side is a concise representation of the whole *Villa Savoye* including four elevations and three floor plans. All of these elevations and floor plans are analyzed using the same procedure, data is produced and represented in the diagram. An arithmetic average is calculated to represent the average complexity offered by the four elevations and a similar procedure is carried out to calculate complexity offered observed in the plan. In this particular case, visual complexity in the facades shows more details in many scales than in the plan.

Finally, average is calculated taking plans and elevations together. Although facades and plans exhibit their complexities in different numbers, it is still not exhibiting the qualitative space in terms of quantity. However based on this knowledge and existing method of analysis further steps can be taken.

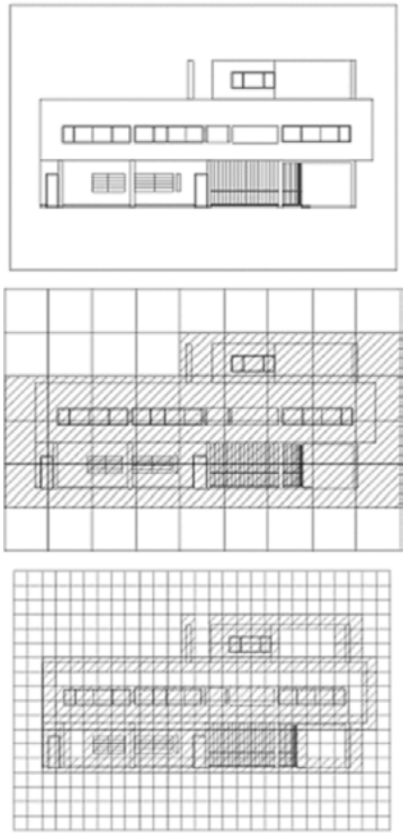

Fig. 33 Villa Savoye elevation placed under grid 1 grid 2 and so on

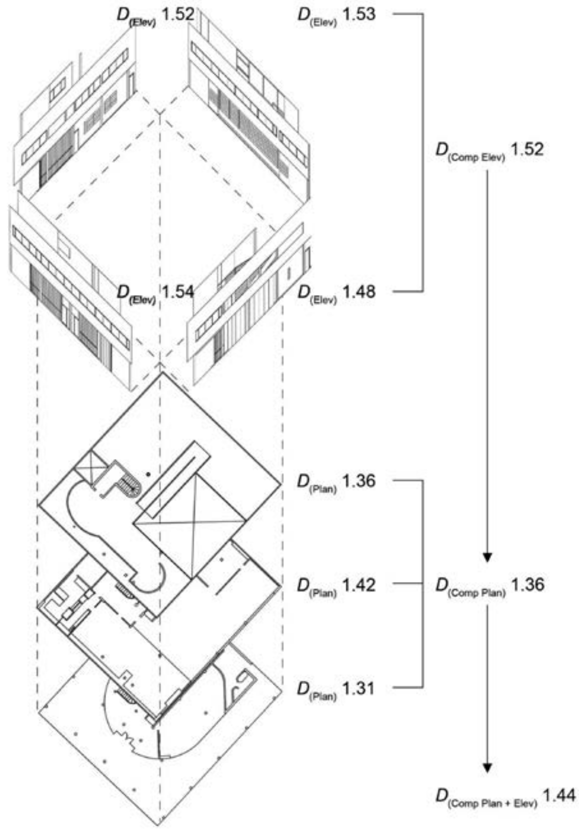

Fig. 34 Diagrammatic representation of different fractal dimension for different plans and elevations

4.3.1 BCM Essentials: Framework and Refinement of BCD

In any research, it is vital to have a framework to provide an analytical finding to the readers effectively. This reflects the *why* and *how* question as important aspects to be reviewed before any research. Here ‚why' question refers to why a building is analyzed or what is the intention of finding out whereas ‚How' question refers to the procedure how such an endeavor is carried out. This how question is vital since it bears the *what* question in it which means what is being analyzed in broader sense whereas what parts, components within a building in a microscopic sense (Ostwald and Vaughan 2016; 67). Buildings and spaces are analyzed to understand the geometric, topological and many other properties that enable designers to design livable spaces. For this purpose several methods have been devised for instance, space syntax research (Hillier and Hanson 1984; Hillier 1996) analyzes habitable spaces with meticulous mapping of lines of sight, accessibility to the spaces whereas shape grammar (Koning and Eisenberg 1981) analyzes the underlying social and functional properties of buildings (Ostwald, Lee & Gu 2017). Fractal analysis is another attempt to quantify the visual complexity of plans or facades which is analogous to Zipf's law or Van der Laan septaves which provide a measure of the distribution of information over several scales (Ostwald and Vaughan 2016; 68).

The following few sub-chapters will discuss how analysts and computational scientists devise a framework for the fractal analysis of architectural work. Based on Ostwald & Vaughan's (2016), *The fractal dimension of Architecture,*[8] which is so far the most detailed work on fractal analysis using box counting method, the basis for the frame work is illustrated as follows:

1. Level 1: Outline

A building skyline or a foot print is taken into consideration since it possibly provides the major planning trends in certain culture.

2. Level 2: Outline+Primary form

In this level, in addition to outline, the primary form or building mass is taken into account which exhibits the characters of design such as major openings like doors and windows, struts, and other major forms. It does not include any secondary forms like brick corbels, minor details that do not satisfy the building mass concept as a whole.

3. Level 3: Outline+Primary form+Secondary form

This level is considered as the building design level where elemental design decisions like

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8 For a detailed overview, refer to Chapter 4 Measuring Architecture in Fractal Dimension of Architecture by Michael J. Ostwald and Josephine Vaughan (2016). pp. 73-85, Birkhäuser, Springer International Publishing AG Switzerland.

mullions of door and windows, rise, and treads, windows panels. Change in materials with one single line and building elements that produce a change in surface level of greater than 25mm must be represented.

4. Level 4: Outline+Primary form+Secondary form+Tertiary form

This level considers the ornamental designs into account that is why it is also called as detail design level. Tertiary forms like doors and window pane, built in furniture, ornamental projections, light fixtures, sanitary details are presented in the drawing. This level exhibits the utmost details of design excluding the textures of the materials used.

5. Level 5: Outline+Primary form+Secondary form+Tertiary form+Texture

This is the finest level where surface finishes, textures, and surface ornament patterns are expressed. The geometry of surfaces like tiles on the floor, patterns in the wallpaper used, applied decorations, floor boards, the texture of the materials such as wood, marble or steel door knobs are taken into account. These details are observed at the close proximity which is not visible from a distance. Since fractal analysis gives details of information across several scales that is why it is considered as an important level.

It is essential to note down that the previous architects and researchers only took up to Level 1 and 2 and sometimes up to 3 into account. Another equally problematic consideration is the graphic representations they used i.e hand drawn orthographic projections like elevations and plans which get pixelated when magnified and affects the box counting tremendously leading to different results. To mitigate such discrepancies, it is essential to generate a standard model or method of calculating fractal dimension. By understanding the underlying problems of fractal analysis in the past, Ostwald & Vaughan (2016) provides two methods to refine the existing technique. They are:

1. Image pre-processing test:

Box counting method is solely based on counting the boxes touching the lines representing the building. If the resolution of the image used, the line thickness and location of the image used in the field (Ostwald & Vaughan 2016; pp.91-93) is not well addressed before the analysis, the results will definitely vary and do not provide the needed information. Therefore, this test addresses those points.

2. Image processing test:

This test addresses two methodological factors which influence the output i.e. the scaling factor, the ratio by which box sizes are increased and grid disposition, the generating point of the grid in the image (ibid p. 87,88). These things are carried out with known classical fractals like Sierpinski triangle, Van Koch snowflakes and so on.

Since the dissertation is related to the theoretical underpinning of the existing method and pointing out the limitations, the descriptions given are taken as the limit.

4.3.2. Use of BCM in Architecture

As of 1990s fractal analysis from urban design to individual building have been conducted by various researchers and scholars. Few of them have already been mentioned in the earlier chapter. Yet some specific and interesting undertakings are described in this chapter. Basically, this section includes three cases of use of box counting method (BCM) in architecture: Urban growth analysis by Batty and Longley's (1994) work in relation to Nikos A. Salingaros's fractal city perspective, accessing urban character by Jon Cooper (2005) and visual complexity of facades and plans by Bovill (1996) and Ostwald and Vaughan (2016). However, there are many other simultaneous researches that have been carried out. Wen & Kao (2005) analyzed and compared the fractal value, using architectural plans, of Frank Lloyd Wright's projects (Harley Brandley House, 1900, Avery Coonley House, 1906-1908 and so on), Mie's Van Der Rohe's projects (Alois Riehl House, 1907) and Le Corbusier's projects (Les Maisons Domino, 1914, Villa Shodan a Ahmedabad House, 1956). Bovill's (1996) hypothesis, „as it is possible to measure the fractal dimension of a site or environment, and then generate a design with the same fractal dimension, to produce a visually coherent addition to a location" (Ostwald and Vaughan 2016; 34) is the starting point to conduct fractal analysis of indigenous buildings and natural forms of Amasya, a historic city, in Turkey and Coastline of Sea Ranch, California which was tested by Lorenz (2003) and Ostwald (2009) with improved technology and refinements in the box counting method. The discrepancy was noticeable and noteworthy (Vaughan & Ostwald 2009). However, the concept of fractal dimension of a site sounds plausible but hard to execute in mathematical terms. Which views will be analyzed to measure fractal dimension and how it will be reflected in the new design are the major challenges. Moreover, the built structure will be of three dimensions, that is why it is highly unlikely for a box counting method to give much information about the space, 3D. Nevertheless there are attempts to use the box counting fractal dimension to contextualize new designs by many scholars like Jiang Liang *et. al* (2012) who used box counting fractal analysis to green space layout in urban square taking research cases in USA, Argentina and China and Jon Cooper (2005), taking Bovill's hypothesis one step ahead, conducted fractal analysis of street edges to understand the urban character of the neighborhood. However, several limitations of the system and the meanings derived from the data were pointed were out.

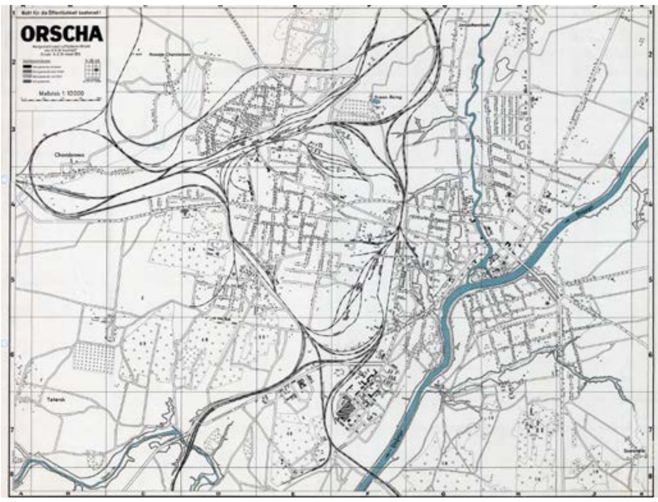


Fig. 35 City Map of Orscha

With the lens of fractal, one can argue, the so called smooth urban fabric is not a smooth. It bears roughness in many scales. An aerial view of the city shows its roughness in its connections i.e. the way roads are connected to each other, how blocks of houses connect with other areas of commercial buildings, from vehicular paths up to bicycle lane and pedestrian paths show irregularity. By coming down closer, blocks of buildings variously sized come into view. Their elevation in the skyline is not smooth neither their structure itself. Irregularity is observed in all directions from a point the observer stands. Regarding a building, it is obvious that it is not a solid cube, it has roughness in many scales on its facades, plans whereas space itself is the hole in the 3D volume. On a reverse thinking, how each object in a room is trying to fill up the hole, space, renders it rough and irregular in 3D volume.

On the importance of scale, as Salingaros (*Unified Architectural Theory*, 2015) points out fifteen properties of architecture proposed by Christopher Alexander to give rise to ‚Phenomenon of life'. Among fifteen, levels of scale' is the first one. This property is the key to understand this roughness observed from an aerial perspective and moving towards the building itself. If one analyzes further this roughness will be seen in human scale which ranges from 2m to 1cm (Salingaros 2015).

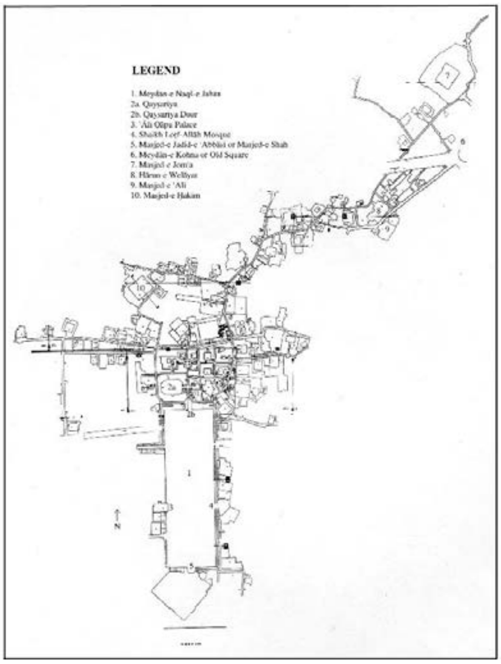


Fig. 36 City of Isfahan, Iran

The hierarchy of scales, from big to small without skipping intermediate scales makes a city coherent. As all the fractals in nature and artificial world show this coherence and hierarchy of scale, it will be an essential tool for urban planners and designers. Cities were studied using fractal analysis since the 1980s from urban morphology mainly taking the streets, transport networks and landscape architecture into account. Yamagishi *et al.* 1988 were the first ones to propose potentials of fractal dimension in urban forms, though Ostwald and Vaughan (2016; p. 58) argue, specific use of box counting method in urban analysis naming the same system as cell-counting method goes to Batty and Longley (1994).

4.3.2.1 Urban Growth & Morphology analysis

> A painter hoping to represent the choppy ocean surface can hardly settle for a regular array of scalloped brush strokes, but somehow must suggest waves on a multiplicity of scales. A scientist puts aside an unconsiouсs bias toward smooth Euclidean shapes and linear calculations. An urban planner learns that the best cities grow dynamically, not neatly, into complex, jagged, interwoven networks with different kinds of housing and different kinds of economic uses all jumbled together.
>
> Porter and Gleick 1990 *Nature's Chaos*

In setting up a relationship between the natural world and artificial world, Porter and Gleick's above quote remains an important analogy of how chaos or irregularity exists from arts to nature to artificial world like a city. The main argument is to shed light on the existence of fractal geometry as much in the artificial world as in the natural. Applications of fractal dimension in architecture started with urban growth and morphology analysis using mathematical tools. In this regard, Michael Batty and Paul Longley's book '*Fractal Cities: A Geometry of Form and Function*' (1994) remains as a pioneering milestone. It was an initial and ambitious attempt to visualize the „complex spatial phenomena" (Batty & Longley, 1994 preface vi). The authors challenged the widely agreed form, layout, and geometry of modern cities with the new geometry of fractals. While introducing how a city grows into a geometric form, the authors posit how the social and economic functioning along with the quality of life within the city plays a key role. In additions, cities are mere reflections of microcosmos of our society and culture which many theories, principles, and ideologies are constantly seeking to comprehend. However physical forms or theory of cities is not easy to relate to its social and economic structures which possess diversity and complexity. On the other hand, the geometry, layout, and configuration of a city are essentials to understand the growth. The emergence of catastrophe theory (Chaos) enables us to understand the underlying complexity and nonlinear dynamics of any system from a local to global contexts like a city, eco system and so on. This changed the whole linearity in thinking i.e cause and effect thinking of the Newtonian world. Moreover, the fractal geometry being part of this paradigm shift in thinking provided a way to understand patterns of the real world that possess hidden order and regularity which seems irregular and chaotic at the first glance. Batty and Longley claims that every city i.e. even planned city shows at least some signs of organic growth which can be studied and analyzed using fractal analysis. The authors compare planned cities which are based on Geometry of Euclid (regular forms) with unplanned organically grown cities which show no such simplicity of forms rather irregular and complex forms. City's morphology and growth are studied based on boundary and area of the different land parcels representing different functions. On the question of how a city is fractal, Batty and Longley (1994) argues,

> Cities have quite distinct fractal structure in that their functions are self-similar across many orders or scales. The idea of neighborhoods, districts and sectors inside cities, the concept of different orders of transport net, and ordering of cities in the central place hierarchy which mirrors the economic dependence of the local on the global and vice versa, all provide example of fractal structure which from the cornerstones of urban geography and spatial economics. (p. 4)

In theory, the descriptions of similar features in many scales of a city hierarchy are likely and have been studied by many urban planners and researchers in the past, however, the approach of defining the form or morphology of a city is debated and criticized. Skeptics such as Mulligan (1997) argued that fractal analysis could not provide any much new information regarding the topics of form and growth. The problem lies in the way analysis is carried out by dividing the whole city into different land parcels, zones and defining their fractal values and later on providing an aggregated fractal dimension of city growth. A city does not grow into different zones separately, but in contradiction as a whole from a tiny parcel of mixed up land uses.

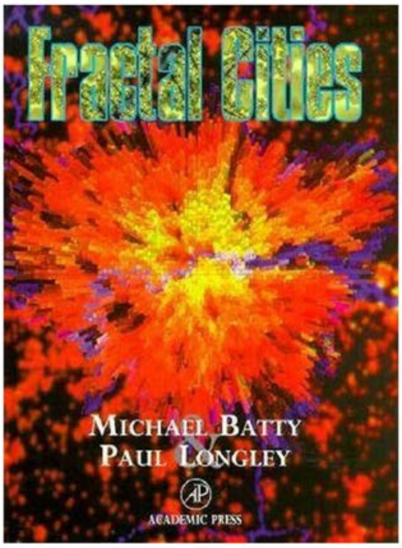
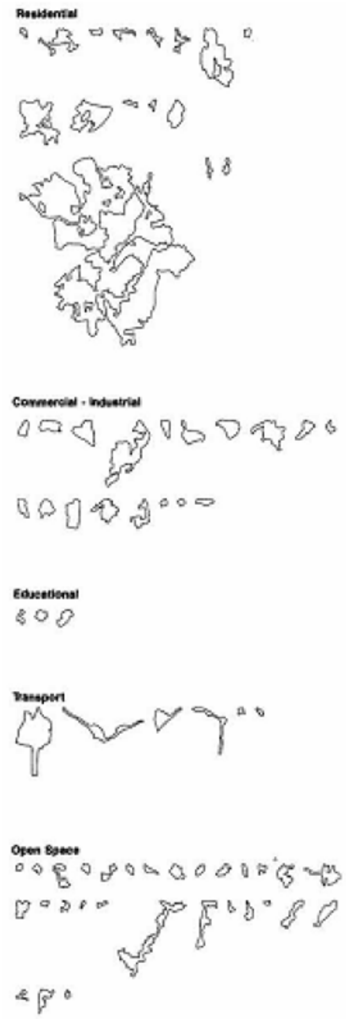


Fig. 37 Book Cover *Fractal cities*, 1994

Fig. 38 Aggregated Perimeter and scale for different land use (Source: Batty & Longley, 1994, p. 211)

Assessment of Urban Morphology

Batty and Longley emphasize the need for new kind of geometry to grapple the notion of organic growth of cities which can not be explained by strict Euclidean geometry, that urban forms must have been developed with underlying theories and conventional wisdom of human geography and urban economics (p 55). Based upon the recurrence of statistical fractal patterns, they based the analysis of recurring patterns in different zones to simulate the urban landscape. Mainly three zones: residential, commercial and open space are taken into account. Mulligan (1997), in a critical overview of *Fractal cities,* posits the use of standard distance based probability functions recapitulates the ‚Hagerstrand´s and Robson´s work on interurban diffusion´ to visualize the city's growth over time. Moreover, the simulations in the book exhibit the patterns of different land uses and overall urban shape based upon density parameters are illustrated to be dependent on the hierarchical recursion. As Mulligan (1997) notices how methods like structured walk, cell counting method were used to estimate fractal dimension of urban boundaries and their growth over a certain period of time. A particular example of Town of Taunton (in Somerset) is modeled and compared with the diffusion-limited aggregation model of physics which means the growth of things around a point. However, real world growth was way more compact than the presented model (Mulligan 1997). No matter how much sound the philosophy of growth is, it is hard to quantify it in numbers and project a sound interpretation of it. However Batty and Longley mentions the problem of finding out the boundary line for a different land parcel for different functions, they again carried out Perimeter and scale to analyze the boundaries and subsequently the fractal dimension.



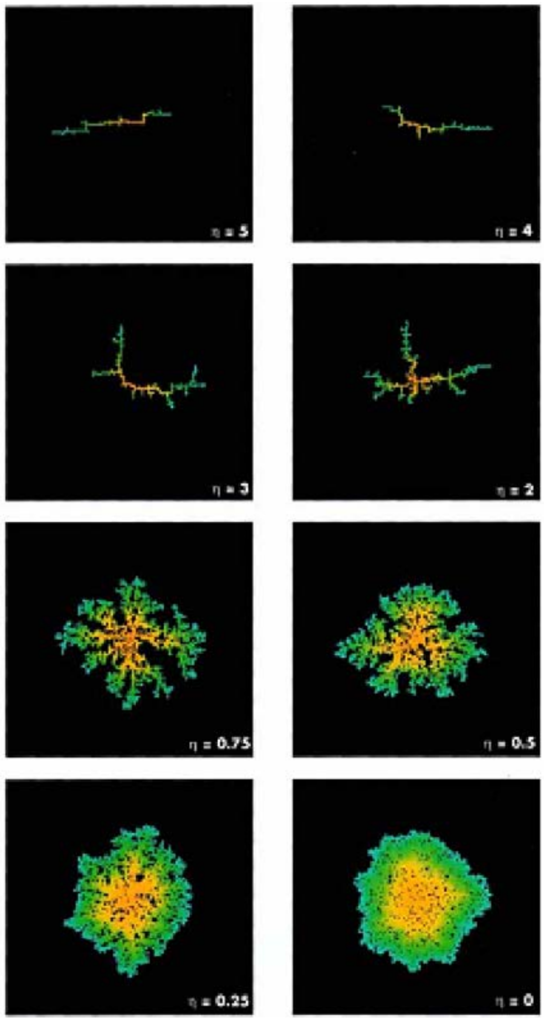


Fig. 39 Separated land use parcel for fractal analysis (Source: Batty & Longley, 1994. p.206)

Fig. 40 Right: Fractal growth of city simulation (Source: Batty & Longley, 1994, p. 74-75)

New Urbanist thinking on Urban structure

(Based on Nikos A. Salingaros's ,*Connecting to Fractal city,* 2003*,* Perspective)

In a constant support of living cities following nature's way of generating patterns, Prof. Salingaros, an important individual in New urbanist movement, argues a city is living when it exhibits the intricate fractal properties through connectedness. On extending the argument, a city's life is ,"directly dependent upon its matrix of connections and substructure, because the geometry either encourages or discourages people's movements and interactions" (Salingaros 2003). This means the importance of connectivity among different nodes of a city is essential to encourage movements, interactions among people, to initiate economic activities, education and so on.

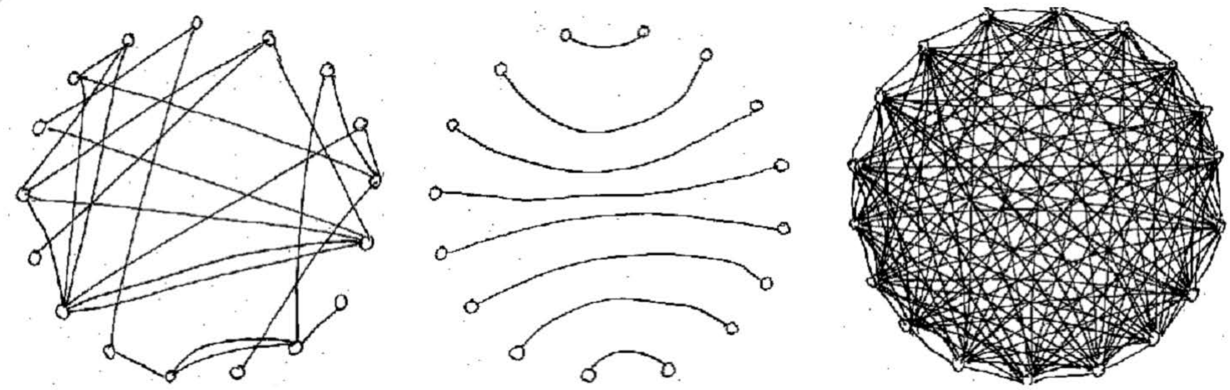


Fig. 41 Left: Randomly connected pairs of nodes, Middle: Pairwise connected nodes, Right: Fully connected city with one node with rest of all nodes (Source: Salingaros, 2003)

The highly intricately connected city shows fully connected nodes (different functions like residence, school, bank, restaurant, parks etc.) without any intermediate node in between. The connectivity depends on the transportation from the vehicular road up to the pedestrian path. It is very important to place a hierarchical order for different transportation networks base on their strength and speed. An order of highways, subways, underground trains, public buses, private cars, bicycle lane and pedestrian path can be considered as a hierarchy from strongest to the weakest mode of connection. Here the strength is directly dependent upon the speed that the mode of transportation provides whereas weakest mode, pedestrian path is where diffusion of human communication is permitted. This is the most fundamental part of creating a fractal network of transportation on urban fabric. The living city exists when these modes of transportation are arranged according to certain fractal quality.

The Fig. 34 is a representation of a hierarchical order of modes of transportation. It leads to the most basic mode i.e pedestrian path where a permeability of information exchange is possible.

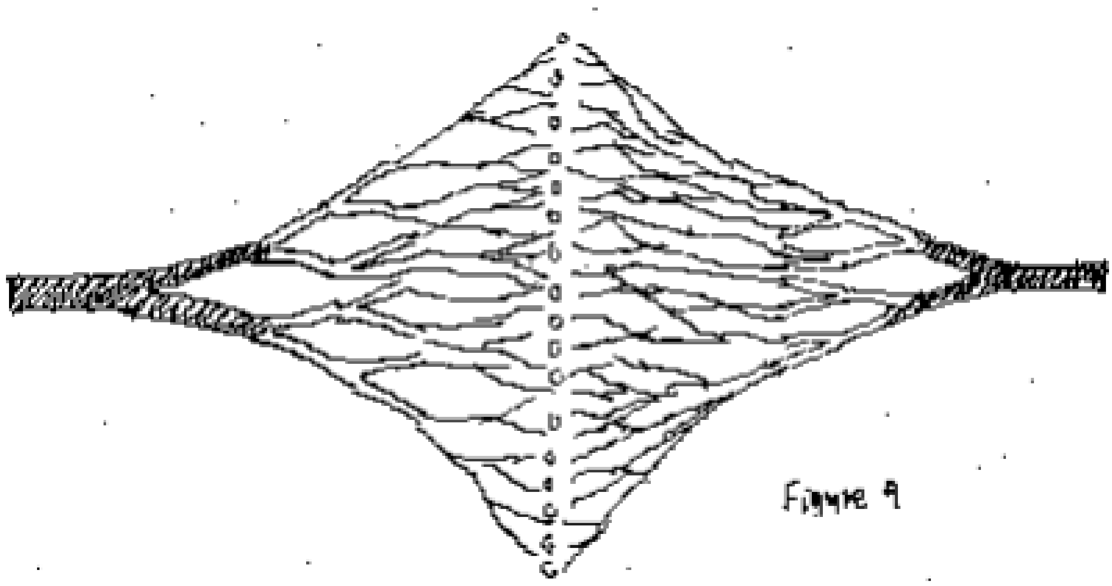


Fig. 42 Crossover requires capillary structure at the lowest levels (Source: Salingaros, 2003)

4.3.2.2. Fractal Analysis in Accessing Urban Character

Considering an extension of Bovill's hypothesis, Jon Cooper's (2005) research on *Accessing urban character using fractal analysis of street edges* is an interesting approach that deals with quantifying the qualitative characteristics of the neighborhood. It means Cooper is trying to understand the visual urban character of a neighborhood. By giving the background of how designers and conservationists tend to record morphological features of a place by plot measurement, block dimensions, and recording facade details' for a reference in upcoming designs in the same context or area, Cooper validates the line of reasoning for the research. This research, as Cooper argues, is similar attempt but using different analysis method of defining a local character which helps designers to 'reflect' and 'respect' with new buildings with a similar character or visual complexity. It sheds light on the notion of 'contextual design' and its importance. The fractal dimension of a context may potentially reflect the underlying characteristic irregularity. The paper examined the fractal characteristics of a series of lines representing the indentation of building facades and gaps along a series of streets. He examined a total of 25 streets in Oxford, England. The scale is used from 14m to 2m and the fractal dimensions were measured using the Box-counting method.

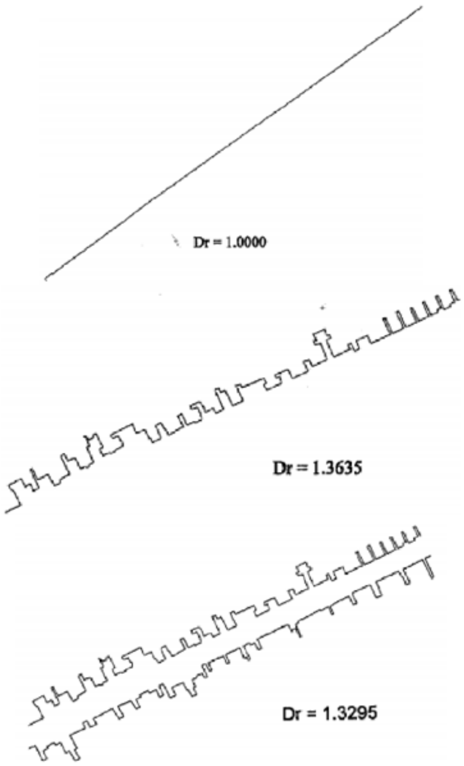


Fig. 43 A comparative study of fractal dimension different street edges (Source: Cooper 2005)

Comparative studies with different fractal dimension values in the figure depict the roughness associated with the street line made out of indentations and frontages of the buildings. However, it is restricted just on the frontages and tells nothing beyond the street lines except few descriptive texts about houses being detached or semi-detached or row housing.

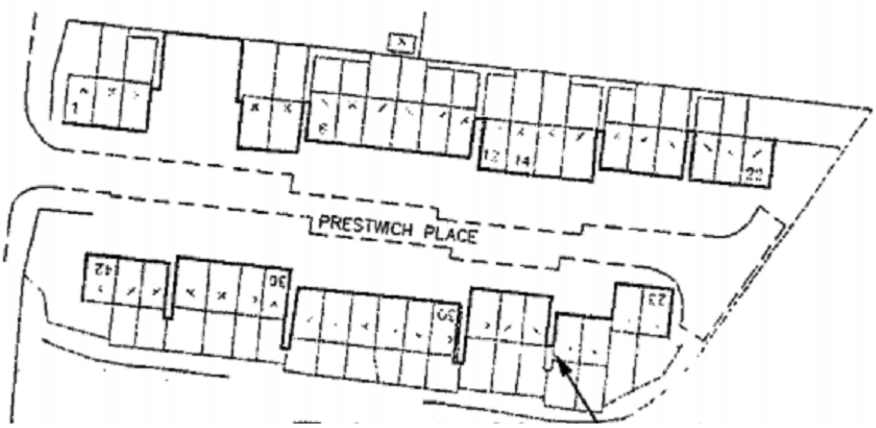


Fig. 44 formation of street edge with building frontages and other built up structures (source: Cooper 2005)

Method for calculation:

The ‚structured walk' or ruler method has been used for the calculation where the distance used for each ‚step' (the ‚detail' of the walk) is related to the scale used. The method possesses one set of dividers or rulers fixed at certain lengths (say s). Then the length of indented street lines are measured and the total lengths (N) is recorded. This allows measurement at various scales(Cooper 2005; 98). By repeating the same method using shorter and shorter stride length with a certain scaling factor, the log-log plot of Number of strides, N in each measurement and the scaling factor is plotted. The slope of the line provided the fractal dimension. The research findings were summarized as follows:

1. Low Fractal dimension signified

   Extensive runs of connected, continuous terraced structures.

   Uniform building sizes,

   Buildings with large frontages,

   Buildings with relatively flat facades,

2. High Fractal dimension illustrated:

   A relatively high number of detached or semi-detached houses,

   A variety of building frontage sizes,

   A low level of repetition in terms of building size.

Although this analysis reinforces the potential of using fractal dimension as a way of 'quantifying the qualitative' which means qualitative characteristic of a neighborhood is quantified in numbers, it still doesn't tell the whole story which is stated in the conclusion of the research paper by Cooper. It does not give much information about the space beyond street edges, it limits our understanding of the visual complexity of the neighborhood fabric. One can logically argue on the urban character provided by the street edges (only with indentations of the building facades) limits the comprehension of the whole neighborhood. It limits the fabric made out of cubes-cuboids (eg. Buildings and other infrastructure) and various different spaces within and around these built structures to just a few lines on the paper. Therefore, this research emboldens the need for higher dimension fractal analysis of the neighborhood for full comprehension where the space beyond the street edges and their third dimension, height is also taken into account.

4.3.2.3. Fractal dimension to analyze visual complexity of facades and plans

After Carl Bovill published his book *Fractal Geometry in Architecture and Design* in 1996 which provided a new way to analyze and to project architectural designs. There were and still are many quantitative ways of analyzing architecture. However, features of any architecture are described in exclusively textual, metaphorical analogies and philosophical underpinning with few graphic presentations. That is why Bovill's undertaking is considered as observing the existing architecture with the new lens. The book addressed the trivial use of fractal analysis in architecture. When somebody observes the facade or a building then information is received in the forms of details like opening details, ornaments, color and so on. The details in several scales are architect's design intention which is reflected on facades and in the plan. The fractal analysis of facades and plans were carried out which gave numerical values known as a fractal dimension. So what does a fractal dimension of a facade or a plan provide?

1. If a façade of a building is measured then fractal dimension provides its characteristic visual complexity. So what does characteristic visual complexity mean? It means the level of detail or formal information that is typically visible across all scales of observation of the facade (Lorenz 2010; Ostwald & Vaughan 2016; p. 3). In the same line, Ostwald and Vaughan (2016; p.142) hypothesizes that fractal dimension of a façade could also reflect the functional qualities of its interior space because possible expression of functions is represented through the location of windows and doors, along with the modulation of walls, roofs, and balconies. However, this hypothesis is not fully satisfied by the results obtained.
2. The fractal analysis of a building plan measures the formal and spatial complexity of a design (Ostwald & Vaughan 2016; p. 142). This spatial complexity is experienced through movement or inhabitation (ibid).

So what's the importance of this study of visual and spatial complexities? The visual experience of spaces as well as spatial experience are the qualitative aspects which are essential yet hard to quantify. Several types of research conducted by Hillier & Hanson in1984 (*The social logic of space*) and by Hillier in 1996 (*Space is the machine*), as cited by Ostwald & Vaughan(2016; p. 142), have demonstrated these experiences of space and form by the inhabitants are reflections of social structure implicit in a building and this property is a significant property of any design.

Digging little deeper in Bovill's book, the most useful and interesting parts are the mathematical calculation of box counting method to measure the fractal dimension of facades of renowned buildings from the past. Bovill analyzed Wright's *Robbie House, Unity Temple,* Le Corbusier's *Villa Savoye*, Alvar Alto's Cultural Centre at Wolfsburg and many other historic buildings. He also compared the organic architecture of Wright with modern buildings of Corbusier and other architects on the basis of visual complexity i.e. interesting details observed on several scales of the building facades. Several researchers, after Bovill, used the similar technique to analyze buildings, landscape, streetscapes, skyline and so on. However, after 20 years, Michael J. Ostwald & Josephine Vaughan (2016) published their book *The Fractal dimension of Architecture* with the refined method of box counting and standardization of minute details taken into consideration while measuring fractal dimension. The book can be argued as the most extensive research in the field of fractal dimension analysis of building facades and plans. However, it still is the fractal analysis of 2D planes like plan and facades and the space, 3D, is lost. On analyzing eighty-five canonical buildings of twentieth century including few from twenty first (1901-2007), with refined box counting method so far, Ostwald and Vaughan (2016) tested few hypotheses. The first hypothesis argues, "*as the complexity of social groupings and functions contained within the home has reduced over time, the fractal dimensions of plans and elevations should decrease to reflect this change*" (p. 4). Although the hypothesis was the obvious outcome of the relationship between social structure and building design expressed in plans and facades, the data provided limited evidence to support the hypothesis and not completely satisfying (p. 370).

Two other hypotheses regarding stylistic genre or movement possessing a distinct fractal dimension and individual architect bearing distinct patterns of three dimensional formal and spatial measures across many scales were also satisfied partially. All these lead to some limitations in our understanding of fractal dimension or its interpretations. At the time it is both likely, I would argue that on behalf of the spatial fractal dimension can elucidate more on the issue.

4.4. Overview

This section provides an overview of the historiography of fractal analysis in architecture world starting from bigger scales like urban morphology and slowly going down to the scale of a single house. With successive descriptions, the limitations are pointed out and why analysis in every scale lacks to project the full application. In case of urban morphology, it is essential to address the points made by Salingaros i.e. to understand how certain aspect like transportation in the previous case becomes fractal and why it is important. After that only, comparative studies between what is a fractal city and what is not can be analyzed. Afterwards, fractal qualities in temporal aspect, open spaces, built structures and so on can be argued and analyzed. In the second and the third cases, the need to answer *The Research Question* mentioned in the *Abstract* is reflected quite significantly. Some thoughts on this issues are put forward in the following chapter.

5 SPATIAL/VOLUMETRIC FRACTAL DIMENSION

5.1. Overview of existing volumetric fractal dimension systems

Several researchers have already pointed out the problems of graphical representations of actual architecture i.e. orthographic views while analyzing using fractal analysis. Few of the solutions are interesting and close to reality. The following observation on Ostwald & Tucker's argument may not be exactly related to volumetric fractal studies, however, it somehow reflects few answers to the research question in the manner that perspective views are actual views rather than parallel projections or orthographic views.[9] Therefore this approach can be considered as a step ahead towards the direction of volumetric analysis. Ostwald & Tucker (2007) and Ostwald & Vaughan (2016; p.233), argue,

> The human eye reads the world in perspective and it is impossible to experience an elevation; the problems of parallax ensure that in the 'real world' no two lines are ever, perceptually at least, parallel. Why not then use perspective views for analysis?

The human eyes perceive the world through a kind of perspective lens which is why it is impossible to read an elevation (orthographic projection) as projected on the drawing sheet. One always sees the lines converging or diverging in the real world. It also relates to Bovill's (1996: 3) hypothesis as cited by Ostwald & Vaughan (2016; 232), „[a]rchitectural composition is concerned with the progression of interesting forms from the distant view of the facade to the intimate details." However Ostwal & Vaughan (ibid; 233) notice that there are several instances in architectural history, this assumption is not taken into account, for instance when ancient Greek applied geometric strategies (like entasis in columns) to artificially correct the visual changes that occur when a building is viewed from different distances and view points, Rennaissance

9 Orthographic or parallel projections are the methods by which architectural forms are represented. Plan (top view), facades (elevations) and Isometric views 3D views (volumetric spaces). For a brief overview [online] available from: https://www.ansatt.hig.no/leifs/orthographic%20projection%20-%20www.helenhudspith.com-slash-resources-slash-graphics-slash-john_h-slash-orthographic.pdf [Accessed: 17.08.2017]

architecture was thought and projected to be admired from one single point of view (ibid; p. 233). The fact that several appreciated architecture and respected buildings may or may not show a cascade of details while approaching towards the building contradicts sharply with Bovill's hypothesis. Yet in case of fractal analysis where one tries to find details on several scales, the argument is clear, however, it is necessary to analyze further steps how perspective fractal dimension can be measured. Tucker & Ostwald provide five variations in alternative framing systems (2007, ibid).

1. Fixed position-one point perspective: The viewer is fixed at a point facing towards the building facade giving rise to only one point perspective and several perspective projections on the picture planes in a predefined straight line perpendicular to the wall are taken into account. Later on, these perspective projections are used for fractal analysis using box counting fractal dimension method.
2. Fixed point-multipoint perspective: The viewer's position is fixed in such a way to have more than one vanishing point, say 2 or more. As in the previous system, perspective projections are taken on the picture planes and are analyzed using fractal analysis. These two systems provide a realistic view but fixing the viewer in place i.e. there is no movement of the viewer.

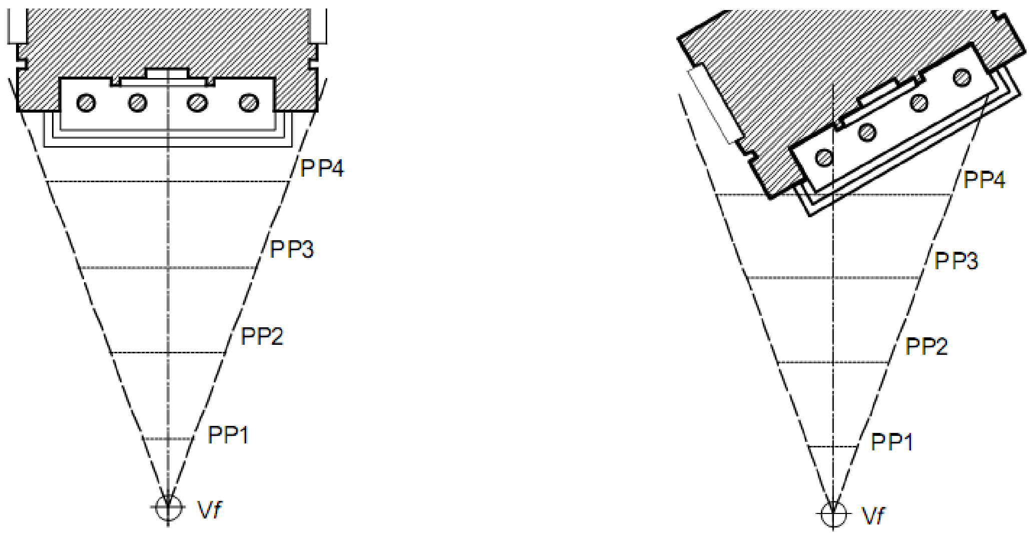


Fig. 45 Left: Fixed position-one point perspective, Right: Fixed point- two point perspective (Source: Ostwald & Tucker 2007)

3. Variable position-one point perspective: It is an improved version of the first system mentioned above where the observer moves along a predetermined path and the perspective projections are taken at different picture planes and analyzed.
4. Variable position-two point perspective: It is the modified version of the second method mentioned above where the observer moves towards a building maintaining the multipoint perspective image. The perspective projections on picture planes taken are analyzed.
5. Variable position –multipoint perspective: This system acknowledges, the importance of human vision' where different positions and different perspectives (from one point to multi-point) are possible during observation of a building. However, a predetermined path is chosen for the observer to move through the building and perspective projections are taken and analyzed using Box counting method of fractal analysis.

Among the five different framing, Tucker & Ostwald conclude 'Variable position, multiple-point perspective' is the closest to reality. In this view, none of the views are orthogonal to the viewer and represents standard cone of vision at each point.

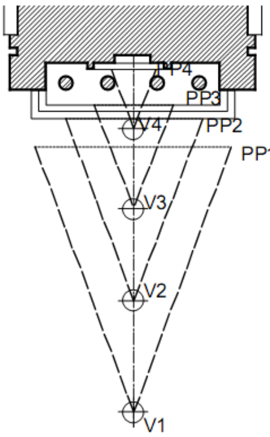


Fig. 46 Variable position-one point perspective

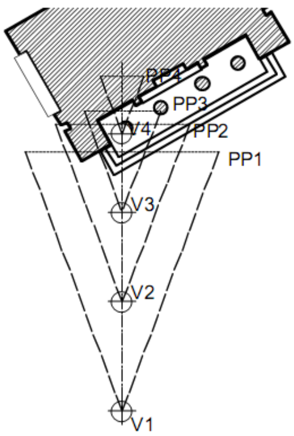


Fig. 47 Variable position-two point perspective

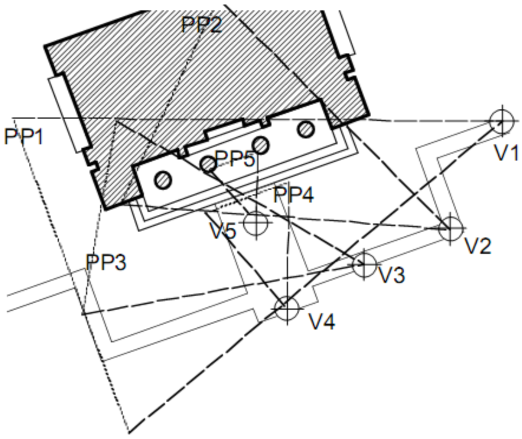


Fig. 48 Variable positions, multiple-point perspective

(Note: V-viewing points, P-picture planes where an image is recorded, Source: Ostwald & Tucker, 2007)

5.1.1. A case study: Perspective fractal analysis of Robbie House

It is important to analyze one of such projects which have been tested using the alternative framing systems argued and presented by Ostwald and Tucker (2007). Later, in *The Fractal Dimension of Architecture´* Ostwald and Vaughan presented an example of *Robbie house* to illustrate how the visual complexity is changing while an observer moves through the building. To standardize the approach so that repeatability of the analysis is possible, authors used Wright´s height to acquire one standard eye level and predetermined path as shown in Fig 49 (Ostwald & Vaughan 2016). Actually the authors were testing one of Hildebrand´s (1991) argument regarding *Robbie House* which is based on *prospect and refuge theory* (Appleton 1975, 1988) that posits a unique spatio-visual experience in Wright's *Robbie House* which is to say „the degree of visual complexity observed while moving into and through Wright´s Building, on average reduces from beginning to end" (Ostwald & Vaughan 2016; p. 238). In addition, the hypothesis tries to corroborate the environmental preference theory about positive spaces experienced while moving through the interior. Therefore, this analysis tests the hypothesis as well as gives a better sense of understanding the space.

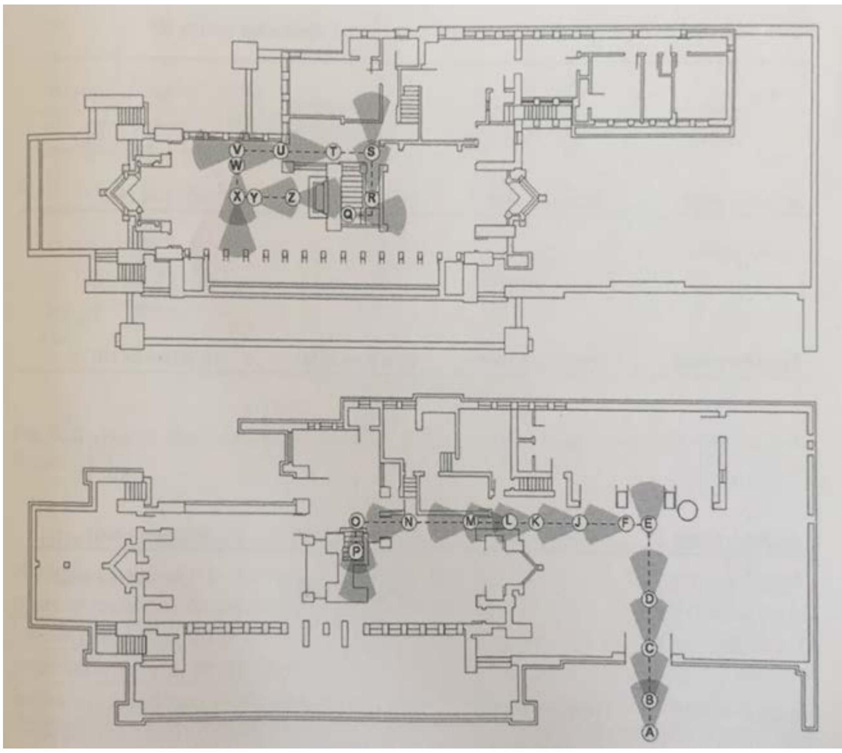


Fig. 49 Perspective route through the ground and upper floors (Source: Ostwald and Vaughan 2016; p. 239)

As shown in the figure, a predetermined path is selected designating from A to P on the ground floor, from entrance up to the staircase. Several perspective views are taken from these specified points and their fractal dimension is calculated as illustrated. For the first floor, specified points from Q to Z are used to take the perspective projections and their sequential perspective projections are analyzed using box counting method and illustrated.

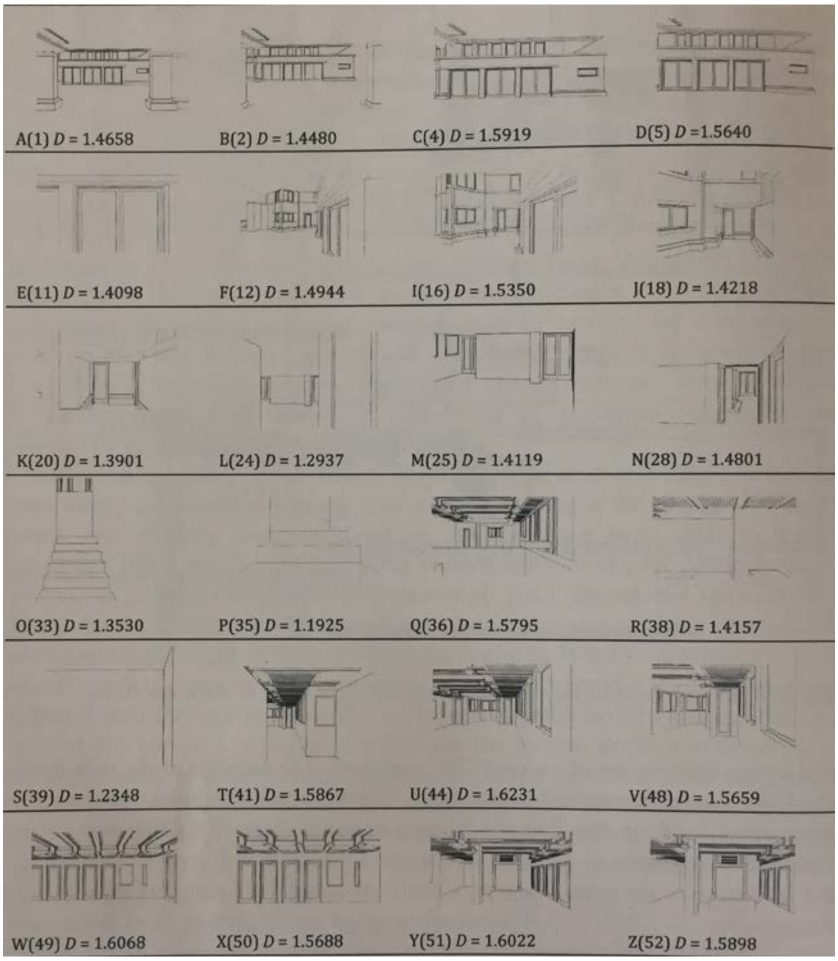


Fig. 50 Perspective projections/views in the Prefixed path and with their fractal dimensions after fractal analysis (Source: Ostwald & Vaughan 2016; p. 240)

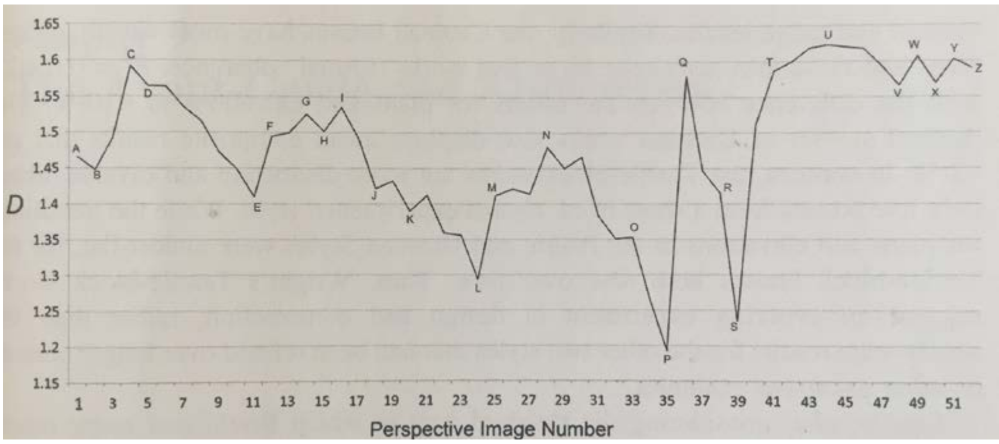


Fig. 51 Fractal dimensions along the path specified (Source: Ostwald & Vaughan 2016; p. 241)

The graph above illustrates how the visual complexity observed during the walk shows the shifting pattern of rising and falling fractal dimensions with different details comes into view[10], however Ostwald (March 21, 2017 email correspondece) argues the repeatability of the method used in future and data obtained needs to be critically analyzed. As of Hildebrand's hypotheis, the analysis and results support his argument (Ostwald & Vaughan 2016; p. 240) i.e the definition of visual of complexity if largely spatial then evidence supports him whereas the data obtained are based on the mouldings, decoratives features which may or may not support the spatial quality as argued by Hildebrand.

Ostwald and Vaughan (2016; 241) mentions the limitation of further speculation on the topic which is the starting point for further research on the need for higher dimension fractal analysis. However, one argument on working out perspective fractal analysis could be a way to measure instantaneous fractal dimension as one sees the view and as one moves around and into the structure, the fractal dimension can be measured constantly and a range of fractal dimension is calculated. This approach nullifies the need for one standard eye level because if fractal dimension changes significantly if the eye level shifts by few inches or centimeters, then there is no certain claim that Hildebrand's argument is right or wrong. This approach analogous to stitching still images together to produce a moving image, one can stitch the fractal dimensions measured in one trip and make a strip of fractal dimension that gives a visual complexity over a range of the views. For Hildebrand's argument to be right or wrong, an experiment with different height people can be carried out giving rise to different strips of fractal dimensions. The average of the result can be of some significance.

On the flip side of the coin related to perspective views being real, one can argue although perspective views are the real projection whatever we see around us, it does not provide that extra dimension or the depth. An image, perspective or orthographic, represents lines, objects in

10 The detailed overview of Robbie House analysis with its minutes details can be found in The Fractal dimension of Architecture 2016 by Michael J. Ostwald and Josephine Vaughan pp. 232-241

space being overlapped which may not be the case in reality. If the observer changes his/her position a little, the overlapped figure may become separate lines which is true in the 3D world. This constant struggle always existed between how reality is presented in 2D graphics and how reality is in actuality i.e 3D world. On the other hand, one can even argue that perspective is not real either because vanishing points are not real, they are just perspective of a viewer or a viewer sees it that way. So which one is true, a cube with parallel sides or a cube with converging and diverging sides with its vanishing points that appear to the observer. Yet one can counter argue whatever one sees is ‚the cube' and that is real. But this is the point, what is a cube? This duality of reality, the perspective ofa viewer and as something (eg. a cube which has equal sides on each face) as it is, questions our understanding of what is what? However coming back to the ground of fractal dimension, this system definitely enhances comprehension of the contested term 'Fractal dimension' with some limitations. Nevertheless, one can argue, why are we measuring 2D fractal dimension and trying to understand space which occupies Volume. Pearson's argument is relevant to this issue as one constantly tries to decipher space (3D) using something which has one dimension less. Architecture is not a mere plan and elevations rather it is the continuous unfolding of the space (3D) into view. Though fractal analysis generating forms and design exists in practice as described in chapter 3 and as argued by Thomas (2012), as cited by Ostwald (2016, p. 34),

> For example… in Gehry's design process, '[f]ractal geometry is applied through programmed formulae in the software and then manipulated to create the resultant form … Instead of forcing conventional geometry onto [a] natural landform, the dynamic positioning of architectural form in context with its site using an iterative design syntax of fractal geometry … will present design possibilities in a meaningful way'.

However, the meanings of such iterative process of generating the form in the landscape are not well analyzed. Gehry's design process is one of such examples where architect takes the whole responsibility of judging what fits and what does not which may not be in agreement with the site itself. Therefore, this sort of application of fractal analysis in architectural design is not taken into account for this research. Since the applications of fractal analysis have been observed after the 1980s in almost every field from physical sciences, biology, neurology and so on, the concept of 3D fractal dimension is around in those fields to cope with the deficiencies of 2D fractal dimensions. Therefore, the concept of spatial fractal dimension is not a new concept it has been practiced in several fields and in architecture as well. Some of the existed 3DFD measurement systems are discussed as follows:

5.1.2. An overview of use of 3DFD in fields of science

Mostly a 3D object is sliced into many small slices and their analysis is carried out using box counting or other methods to find out the problems in the object. But full comprehension of the object seems lacking as 2D fractal analysis is incompetent to provide full information. In the case of the biology of bone tissues, structures, and function, it is necessary to analyze volumetrically to know what is the problem inside the bone. Akari *et al.*(2008, p. 48) situate the need for volumetric analysis of Trabecular bone tissues. They argued, "[slice imaging] depend essentially on the directions of the planes and so there is- in general case- no relation between 3D and 2D fractal dimensions." This critical argument clarifies the false known assumption of the 2D fractal dimension being the representation of 3D volume. There is no clear relationship exists between these two fractal values. Though the opinion is of medicine field, it clearly justifies the argument of understanding three-dimensional volume is only possible through volumetric analysis no matter whichever the field is. Wahl (1995) illustrates the fractal dimension measurement of volumetric space using boxes (Lattice boxes in space) rather than conventional box counting method on flat plane.

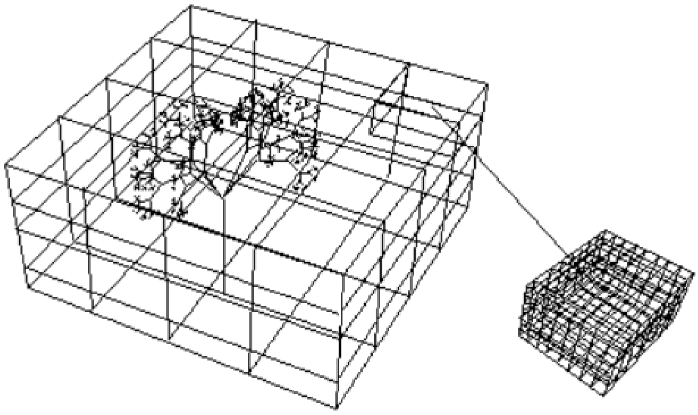


Fig. 52 Cube-counting method for a volume, a 3 Dimensional Lattice

This mode of analysis somehow addresses the Pearson's argument stated above. However, it still lacks the interior space which the concern of this research. It could have been more effective if one could have analyzed the internal features with a sectional 3D fractal value of a section which helps to understand the interior space. Wahl (1995)[11], describes volumetric dimension calculations is similar to the planar box counting method. One just uses boxes (Length, Breadth, and Height) in place of Tiles (length and breadth). The 3-dimensional volume is placed inside the array of lattice boxes. Wahl posits," ... by counting the boxes containing at least part of the object a ratio is established between the box size and its corresponding count. This ratio at different scales determines the object's dimension". Suzuki (2007) argues the similar point with an analysis of 3D tree models. He posits, „The fractal dimension FD of a set of voxels is expressed by FD = log(Nr)=log(1=r)" which is exactly similar to the box counting method with just one addition of the third dimension. The process includes putting the model into a cube As in the figure.

11 Wahl, Bernt (1995), Chapter 4 Calculating Fractal dimension, [online] Available from: http://www.wahl.org/fe/HTML_version/link/FE4W/c4.htm [Accessed 15 June 2017]

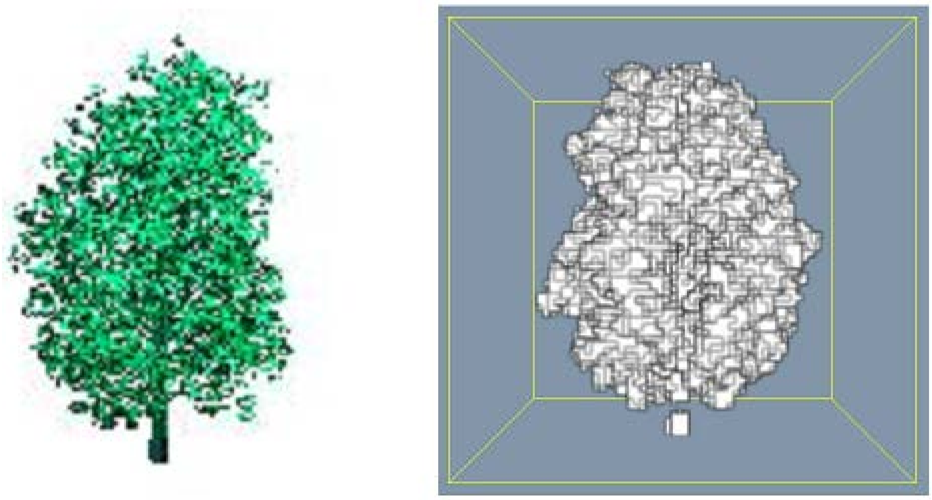


Fig. 53 Right: A Tree model, Left: Voxelized model (Source: Suzuki 2007)

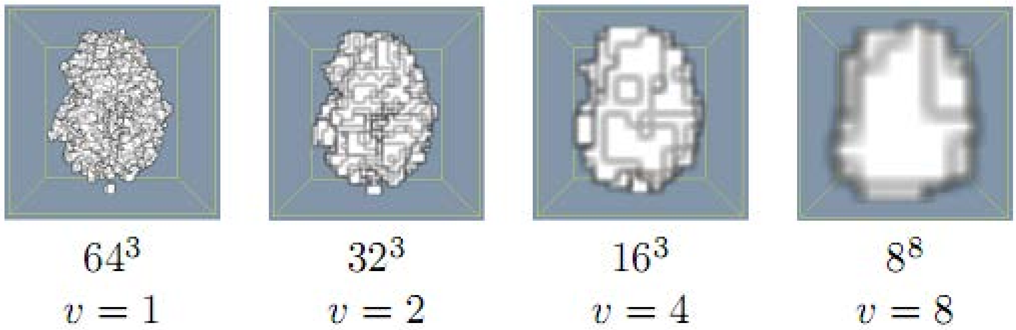


Fig. 54 Voxels of different scale (Source: Suzuki 2007)

Suzuki (2007) explains, „Next, the 3D polygonal model is voxelized with sizes of 64 X 64 X 64. During the voxelization process, a value 0 or 1 is assigned to each voxel. The bounding cube is broken up into tiny unit cubes with sizes of 2X2X2, 4X4X4, 8X8X8, 16X16X16 and 32X32X32.The process creates multiple resolutions of voxel data." The values of 0 represent those voxels/cubes that do not touch the tree model's any part whereas value 1 represents those voxels that touch at least some part of the model. Other than that, the process of making small cubes as scaling down the initial bounding box is the exact same procedure done in decreasing the grid sizes in box counting method. After the cubes/voxels are counted, the log-log plot of cube sizes (1/r) vs the number of cubes touching the model is graphed.

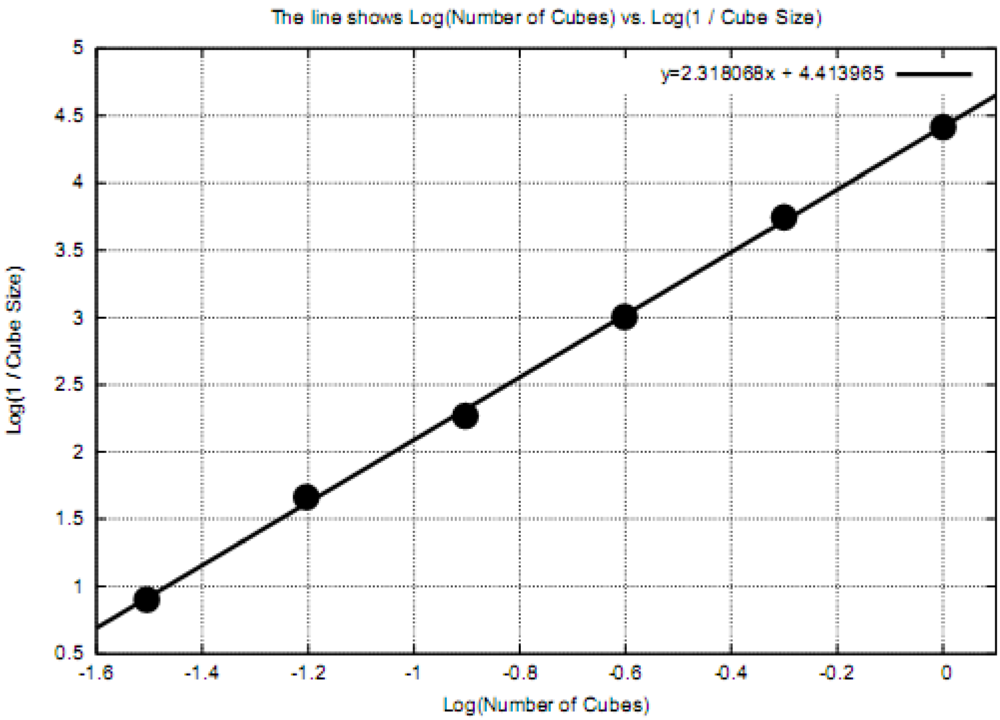


Fig. 55 Log-log graph plot according to the data obtained (Source: Suzuki 2007)

The slope obtained gives the 3D fractal dimension of the model being analyzed. Suzuki (2007) explains, rougher the cube higher will be fractal value reaching nearer to 3. A similar procedure was described in Villoladas *et. al* (2011) paper on calculating 3D fractal dimension from MRI image data to model the entire volume of brain that helps in „characterization and quantification of the morphology of the brain" (p. 452). In this case slices of brain, images are attached and voxelization is carried out. It is an effective tool to quantify brain morphology.

5.2. Reflection on Research question

5.2.1 Cube Counting Method

It is the extant and upgraded version of box counting method and many of the fields have used this method for analysis in the name of voxelization of the model being analyzed and counting the voxels/cubes as we count boxes in the box counting method. It can be applicable in the context analyzing urban character; it analyzes the whole neighborhood considering each building as cubes or cuboids providing a tentative massing of the context. Possibly, the shortcomings of Cooper's research i.e. not addressing the urban character beyond the indented street lines, can be remedied in a collective analysis of fractal dimension of indented street line and cube counting dimension of the neighborhood. Cubes on different scales are counted to find out the 3D fractal dimension of the whole neighborhood.

5.2.2. Sectional Strip analysis method

This was suggested by Prof. Jon Cooper during my personal communication. A building's external fractality analysis is the attempt in this method. This method is similar to single line study of street edges with one change, a builing, whose analysis has to be done, is cut horizontally and vertically and then „unfolding the slices to create a series of single long lines that encapsulate the projection, recession and fluctuations of the buildings external contours" (Cooper 2017).

Then it is analyzed similarly to that of Cooper's strip measuring the fractal dimension of external contours of the building.

5.2.3. Un-Folding Space method

As the theory of fractal suggests, fractal dimension is a space filling properties. A jagged line trying to be a plane but not yet a plane shows fractal dimension between 1 and 2 i.e. the analyzed line is neither in one dimension nor in two dimensions. On the other hand, a plane with holes carries a dimension between 1 and 2 i.e. the holes created on the plane makes it fractal (less than 2-dimensional) from a 2-dimensional plane. One can form an index of coherence' (Lorenz 2010; 3) from a strictly Euclidean shape like a smooth rectangle and then start adding holes in it (say, doors, windows, and other penetrations, detailings) one by one. This shows how fractal properties appear on a surface. As shown in the Fig. 46, it is a method of reversing the dimensional value of a plane possessing a dimensional value of 2 and while puncturing holes in it, one constantly makes it less likely a plane rather something in between a line and a plane.

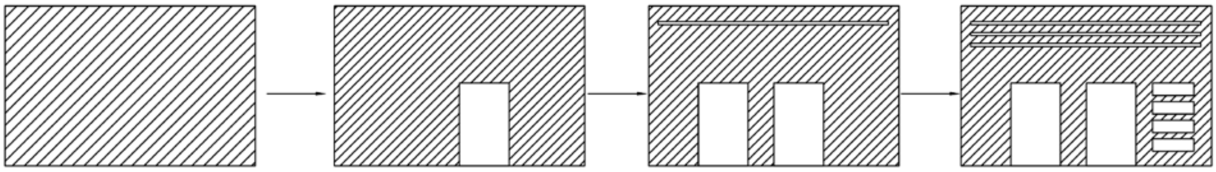


Fig. 56 Index of coherence: from left to right: A rectangular plane (Euclidean), A Penetration as a door in the plane, Two doors cut-out along with top penetrations and Other holes as design elements (Higher degree fractals)

On the higher dimension, the space is composed of these surfaces when placed together giving a sense of closure. The simplest enclosed space of all is the four walls, a roof, and a floor. Yet this space itself is not a cube which is a Euclidean 3 Dimensional object. So the question is, what is the dimensional value of this space? One can move in three directions but it does not represent the three dimensionality. This means the whole space itself is a hole in the 3 Dimensional world similar to the hole we observed in the 2-dimensional plane. Moreover, in reverse thinking, this hole is filled up with many objects like furniture, lighting fixtures, swinging doors and windows, kitchen cabinets, the partition walls and so on. The most interesting part is that the human is also filling up this hole, the space, like other objects. It means as a person moves in a room, the pattern of the hole constantly changes probably rendering different 3D fractal dimensions. This filling up of the hole leads nearer to the 3-dimensional world yet not fully 3 dimensional. These objects try to fill up the hole created by the space tending to make it a solid 3-dimensional cube. Thus this is the starting point where the notion of spatial dimension measurement begins.

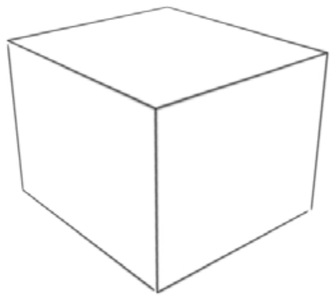


Fig. 57 An enclosed space with four walls, a roof and a floor.

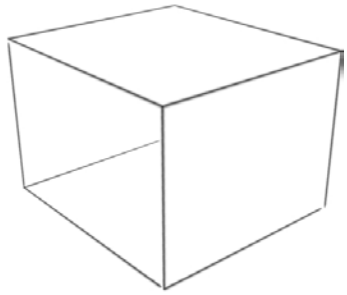


Fig. 58 A space, enclosed on three sides and opened up in one.

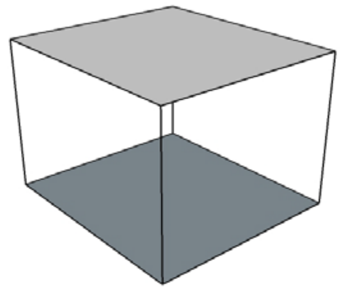


Fig. 59 A space made out of roof and a floor.

This method involves this observation of space created through unfolding of the planes and reversing back the process. Analogous to the box counting method of facades and plans that try to analyze the properties of different lines or holes that make surfaces rough, Un-Folding space method analyzes the hole, the space, creating planes along with its design details and furniture which makes the 3D space rough. All the walls and roof are unfolded to have a flat plane. This flat plane bears roughnesses; that makes the **space** rough, at many scales.

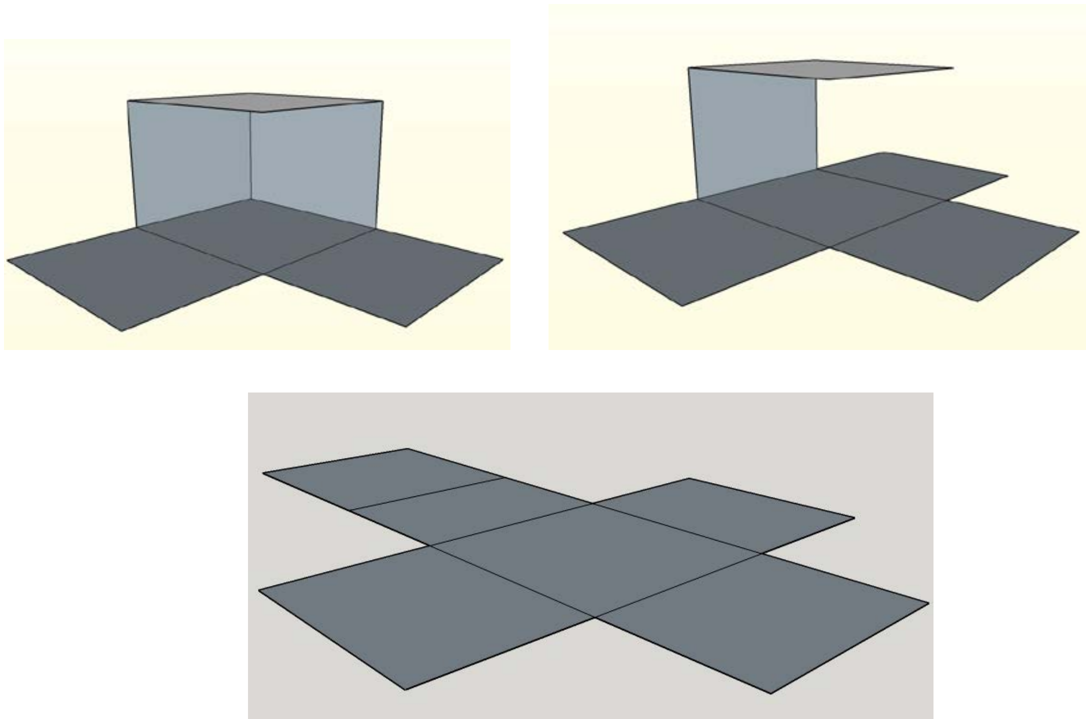


Fig. 60 The stepwise process of Un-Folding of a cube

The present day popular way of calculating 3DFD is by creating voxels and counting the voxel numbers at different scales. The whole process is termed as voxelization. The similar procedure is carried out on the unfolded space which is a rough flat plane. The scaling factor can be taken as 1/3, which exhibits the most naturally occurring fractals like cauli flower and so on (Salingaros 2003), from the starting point. It means every time the size of boxes will be reduced by one-third and then cube counting is carried out. The log-log plot of the number of cubes counted vs the scaling factor as in the Box counting method will provide the 3DFD of the surface being examined.

However, for the next part the unfolded space is folded again and then voxelization with similar scaling factor is carried out. The graph plotted now will provide the 3DFD in its cubic form. The comparative analysis between these two 3DFD values is a crucial point as both of them represents the same space but in two forms: Unfolded and Folded. Further research of the data obtained and its interpretation will be an important contribution to the present research. This is just an initial attempt to provide a way of space reading in its two forms. The correlation between these two forms of same space can be studied with different mathematical tools. On the interpretation part of the numerical values i.e 3DFD value, one conventional method of interpretation is obtained by comparing the spatial complexity (in volume) with the visual complexity (in facades & plans) in Box counting method across many scales. This provides the literal meanings behind the numerical values obtained. However, this literal meaning may or may not be applicable in designs or architectural works. That is why I would propose one experiment as the further research in the same line.

Further Experiment

Two separate rooms are created: one, let's say room A, with smooth wall, roof and floor with no textures or design, even the door and windows are textureless, the other, let's say room B, with usual design intentions like textures and design on the walls, windows and ceilings and usual floor patterns. The fractal dimension of the sapce is calculated using the Un-Folding Space method.

In the second part, participants are divided into two groups. One is sent to Room A and another group is to Room B. Now their stress level being inside these rooms is checked using various technqiues like saliva test, skin conductacne, heart beat and so on while the participants are given some metal tasks like arithmatic numerical problems. The result of the stress level before and after the experiment will be the effect of the space, 3DFD, on them. On the extension of this experiment, many rooms with different fractal dimension (3D) can be created, let's say 3DFD value ranging from 2.1 to 2.9, and participants' stress levels are checked as in the previous experiment. This will show the relation between the spatial fractal value and its effect on the inhabitants. It may also elucidate, with effective test framework and methodology, the psychological as well as physiological effect of different spatial configurations on human brain. This may help designers to design accordingly.

6 CONCLUSION AND DISCUSSION

The curiosity to know *the space,* where we are, has been in our brain from time immemorial. Philosophers, thinkers have explained and have understood it in many different ways, mathematicians and scientists have explained in many other ways whereas architects have designed and created it for living, growing and thriving. However, there is no full comprehension about this space and researches, in the field, are still trying to understand. The space is perceived through our senses, body, brain (logic) and may be something beyond (consciousness). While elucidating the importance of perception, Bovill (1996) writes,

> Perception is a complex process. Our senses record; they are analogous to audio or video devices. We cannot, however, claim that such devices perceive. Perception involves more than meets the eye: it involves processing and organization of recorded data. (p. V, Foreword)

Understanding architecture is as much complex as understanding perception. Buildings are the expressions of our social, economic, cultural and many other aspects of the society. Ostwald and Vaughan (2016; p.2) argues, „Architecture is not just space and form divorced from purpose, geography or human aspirations", however the space and its relation with geometry can not be overshadowed as well. The very space where human resides, is created with certain geometry. Going back to the perception and recorded data again, basically this data is what every analysis tries to capture and attempt to define the perceived world. What do we record and process as we see is an interesting phenomena. Lorenz (2002; p. 31) posits,

> From a physiological point of view man can take up a total set of information of 10^9 bit per second out of the rich perception offered. 10^2 can be dealt with in our consciousness, but only one bit per second is saved in our memory!

This information is what fractal analysis deals with. Ostwald & Vaughan (2016) posits," [a] fractal dimension is a rigorous measure of the relative density and diversity of geometric information in an image or object" (p. 3). Therefore, it determines the amount and distribution of information dispersed across many scales. In case of architecture, those informations are mere lines, different shapes, voids and so on. However, in conventional qualitative analysis, geometry, particularly dimensional analysis, reveals many aspects of any design, culture, history. The philosophical foundation provides context for the design to situate in particular site and architect's understanding of the site and client's emotions and expectations are expressed in the final design.

Fractal analysis is the approach that touches the untouched portions of these information in numerals yet it lacks logical interpretation. That is why, more research, in the field, is needed. The concept of *Space* being the hole in the 3D cube is analogous to the holes in a plane making it rough and reversing back the process of Un-Folding is the point elevated in this dissertation in particluar. The word *hole* seems trivial and meaningless at first instance, however it is the whole subject of fractals. How holes are trying to fill up the uncovered space or in opposition to reduce the covered/filled up space, both of them are fractal nature of the surface, being affected by the *hole*. It determines how much fractal or rough the surface will be. The space, defined as a *hole,* plays the same role i.e making the 3D rougher. How the space is designed? how the 3D volume is being affected with this *hole?* renders the volume *fractal*. One interesting phenomena is, a person in the 3D space acts as the space filling object. In the reverse thinking, a person is changing the fractal values constantly as s/he moves around. This makes the seeing visual complexity less likely when talking about spatial fractal dimension. Rather spatial fractal dimension is how constantly the spatial fractal value changes with certain fixed space filling objects like fixed furnitures and one dynamic (constantly moving) space filling object, humans. That is why this contemplation makes this research more qualitative than quantitative. The spatial dimension is correlated with experience rather than visual. That is why its interpretation is never straight forward.

2D fractal dimension and 3D fractal dimension may affect each other in some aspects but understanding 3D volume with 2D analysis makes it less reliable. However, on the informations part, gained after 2D fractal analysis is important. The interpretation of the dimensions obtained is another equally important yet challenging section of the analysis. In my opinion, comparitive study of past architectural movements and practices with fractal analysis to show any particular trend or signature fractal dimension is less important. Moreover, the analysis should provide better conditions for whom it is designed for. By saying better conditions, I am referring to the psychological, physical, spiritual and every sort of well being of the inhabitants. That is why further research must be concerned with the interpretations of the results obtained from fractal analysis and these results must be rigorously checked with experiments like the one explained in the *Un-Folding space Method.*

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